Economic Theory (2025) 80:619–658
https://doi.org/10.1007/s00199-024-01583-8

RESEARCH ARTICLE

Feudal political economy

Desiree A. Desierto[1] · Mark Koyama[1]

Received: 22 July 2023 / Accepted: 15 May 2024 / Published online: 10 July 2024

Abstract
How is order achieved in a realm in which every elite commands both economic and military resources, and no stable institutions of power exist? We depict coalition formation in the feudal world as a series of non-binding agreements between elites who can move in and out of the coalition, through peaceful and violent means. We derive conditions under which the realm unites under one rule — a grand coalition, or remains fragmented. We motivate our analysis with key historical episodes in medieval Europe, from the Frankish Kingdom in the 5th to 10th centuries and England in the 11th to 15th centuries.

Keywords Political economy · Bargaining · Coalitions · Feudalism · States · Conflict

JEL Classification C72 · C78 · D74 · N43

1 Introduction

Absent institutions governing the use of power, how is political order achieved among elites that each have their own economic resources and independent military capability? Examining such a "feudal" environment, we show how a ruling coalition emerges from a series of elite bargains, enforced through peaceful or violent means.

Traditionally social scientists presumed the existence of a functioning state; that is, a state defined as a political entity possessing a monopoly of legitimate violence within a given territory (Weber, 1968). But such states are a historically recent phenomenon

We are grateful for comments from Scott Gehlbach, Cesar Martinelli, Jean–Paul Carvalho, Jared Rubin, and from audiences at ASREC (Chapman University), the PPE Seminar at George Mason University, and the University of Pittsburgh, including Leticia Arroyo Abad, Pete Boettke, Jenny Guardado, Vincent Geloso, Erik Kimbrough, Mike Makowsky, Avital Livny, Noel Maurer, Ilia Murtazashvili, and Larry White and others. This work is funded by Grant 62593 from the John Templeton Foundation and a Mercatus Center's Pluralism & Exchange Grant.

✉ Mark Koyama
mkoyama2@gmu.edu

Desiree A. Desierto
ddesiert@gmu.edu

1 George Mason University, Fairfax, USA

(seeStrayer, 1970; Tilly, 1975, 1992). During the medieval period, and particularly between 900–1200 CE, much of Europe was governed by feudal polities. Spruyt (1994, 34–36) characterized such feudal polities as "a highly decentralized system of political organization which is based on personal ties." Because violence potential was decentralized, these societies confronted what North,Wallis and Weingast (2009) call "the problem of violence".[1]

Several important papers that study the emergence of the state from anarchy include Olson (1993); Moselle and Polak (2001); Bates, Greif and Singh (2002), and Grossman (2002). There are also formal models on aspects of the transition from weak extractive states to inclusive institutions (Myerson, 2008; Acemoglu and Robinson, 2023). These models capture the importance of monopolizing violence, but do not explicitly depict bargaining between elites. Our model captures the idea that elites can bargain as well as fight and hence allows us to characterize a feudal political order as one that involves both cooperation and conflict. We use this framework to examine the conditions that led a feudal polity to be either consolidated or fragmented.

In a recent contribution, Levine and Modica (2022) develop an evolutionary model of conflict which they apply to study the historical rise of Western Europe. Building on Levine and Modica (2013), they analyze how conflict between a commercial and a military elite can drive institutional evolution. Where the latter dominate, extractive institutions prevail. In contrast, where the former prevail more inclusive institutions are viable. A key feature of this framework is that it is attentive to the balance of power and to whether or not the overall state system is characterized by societies with inclusive or extractive institutions. In their model, the threat of outsiders and the defensive/offensive capabilities of the prevailing military technologies play critical roles. A balance of power between inclusive states requires the threat of outsiders to be strong and the defensive fortifications to be relatively weak. Levine and Modica (2022) find support for these predictions in the history of Eurasia.

Our focus in this paper is not on the ascent of Western Europe but on characterizing the basic political economy of feudal societies prior to this rise. We construct a bargaining game played by the elites of a territory, in which an aspiring ruler proposes an alliance to every other elite. Under an alliance, the elite commits all her resources – economic and military, to the ruler's coalition in exchange for a share in the coalition's total resources. This commitment, however, is non-binding, as any member of the coalition can subsequently rebel and seize back what she can from her initial contribution. If the elite rejects the proposal, the ruler attempts to force the alliance through battle, aided by some key members of her coalition. Borrowing from Ray (2007), we call this group the "approval committee". As the game is infinitely repeated, the coalition expands whenever a player joins, peacefully or through conquest, and contracts whenever a player rebels.

In equilibrium, either the realm is consolidated into one grand coalition, or remains fragmented. The key determinants are the fighting costs of individual elites, their resources, and the extent to which these resources are appropriable or non-appropriable

[1] The importance of the problem of violence is also central to the analysis of Bates (2001); Bates et al. (2002); Bates (2017), Barzel (2002), Cox et al. (2015) and Weingast (2020).

by the ruler — where non-appropriable resources are those that can be captured by rebels and safe guarded from the ruler.

We obtain several results that shed new light on our understanding of feudal political economy. Specifically, we show that many intuitions we have for thinking about the conditions that favor political consolidation in centralized modern states do not apply in a feudal environment.

First, we show that when resources are large, consolidation is more likely. While this is a standard result, in a feudal environment it occurs not because those resources can be used by the ruler to centralize power, but rather because they can be distributed to elites to entice them to join, and stay in, the coalition.

Second, we find that the more appropriable these resources are, the *less* likely is consolidation. This prediction may appear counterintuitive – one might think that when a ruler can easily appropriate resources, she would become powerful enough to consolidate a realm. Note, however, that when resources are appropriable by the ruler, anyone who rebels can then only seize a small amount of resources from the coalition. Rebellion is thus less costly to the ruler, making her more willing to suffer rebellion rather than distribute resources in order to prevent it. Rebellions are thus more likely to occur, and consolidation less likely.[2]

Third, we show that fighting costs matter, but specifically only the fighting costs of the weakest members. If the weakest elite is sufficiently powerful that the ruler wants her to join and stay in the coalition, then the ruler will want everyone else who are stronger to do so. Similarly, only the fighting cost of the weakest approval committee among all committees formed over time matter. When this weakest group is powerful enough that the ruler is willing to prevent them from rebelling, then she will also prevent rebellion from any stronger set of members. Thus, if even the weakest fighters are valuable to the coalition, the ruler is always willing to bargain and share resources to keep everyone in a single, grand coalition.

Finally, our model can explain why castles played an important role in the consolidation of feudal polities. Castles enabled rebelling feudal lords to better defend nearby agricultural lands against the king. In other words, castles made land non-appropriable by the ruler. The results of our model, therefore, imply that castles would have facilitated consolidation, and in fact they did. A feudal ruler would be less likely to want to lose a lord with a castle – his rebellion would be more costly to the ruler, which makes the ruler more willing to bargain or share resources to prevent rebellion. In equilibrium, the realm is consolidated. It is only in the early modern period when power became concentrated – that is, when rulers no longer relied on nobles' contributions and realms ceased to be feudal, that castles became an impediment to consolidation. They emboldened nobles to rebel, but the ruler could simply fight and crush them, rather than bargain. Eventually, rulers were able to dismantle and destroy the castles to finally overcome these barriers to consolidation.

[2] In contrast, the civil war literature (see, e.g. Fearon and Laitin (2003)) shows the opposite – when rebels can easily capture and keep resources, rebellion is sustained and consolidation is unlikely. This is because a state is less likely to bargain with rebels, whereas a feudal ruler is more dependent on other elites and therefore is more willing to share resources to prevent rebellion. However, she would only do so if the exit of rebels would be sufficiently costly – precisely when rebels can easily seize the resources of the ruler's coalition.

Our work adds to the formal literature on coalition formation (see Ray and Vohra, 2015). We borrow from Ray's (2007) proposal-based model of coalition formation with non-binding agreements.[3] Starting from a state in which individual players are fragmented into several coalitions, a player — the proposer, offers to another player — the responder, a new state in which the latter is included in the former's coalition, which the responder can accept or reject. However, any move to a new state, which changes the composition of the coalition, has to be approved by an approval committee, which is a subset of the proposer's coalition. Members of the approval committee may not approve the proposal and exit the coalition. In this manner, their previous agreement with the proposer to join/stay in the coalition is non-binding. Ray (2007), however, does not specify the process of non-approval and exit. In modeling coalition formation in the feudal context where violence plays a dominant role, we interpret exit as rebellion and, in addition, include the possibility of future agreements being forged through conquest. Our model, then, is one of coalition formation with violent entry and exit.[4]

Our paper also contributes to the study of feudalism as an exemplar alternative form of political organization compared to the modern state. Understanding it better provides insights into how order can be maintained in the absence of a monopoly of violence. Indeed, social scientists from Smith (1776) and Marx and Engels (1848) to Moore (1966); Anderson (1974); Tilly (1992) and Ertman (1997) have seen the importance of the transition from feudalism to both a market economy and the nation state as a critical stage in the emergence of modernity.

A smaller number of papers have explored the distinctive political economy of medieval polities.[5] Chaney and Blaydes (2013) document a divergence in the duration of rule in Europe and the Middle East after 800 CE, which they attribute to the stability provided by feudal institutions that encouraged bargains between powerful nobles and the monarch.[6] Leon (2020) models the size of a ruler's coalition in medieval England. Studying a game comprising three types of players: the King, the barons, and the peasants, he shows how the threat of rebellion can induce the King to grant rights to elites. His framework differs from our model as it does not involve coalition formation. Salter and Young (2023) argue that the polycentric sovereignty characteristic of feudal polities laid the foundation for the emergence of representative and constitutional government.

[3] Koutsougeras (2022) in contrast studies coalitions in a market setting using the tools of cooperative game theory. Okada (2023) uses non-cooperative game theory to study bargains with voluntary participation and renegotiation but not in a coalition formation setting.

[4] A related model is Acemoglu, Egorov and Sonin (2008) who analyze the stability of coalitions in non-democracies where there are no institutions that assign political power but, rather, individuals are endowed with political power and are free to combine their endowments by forming coalitions.

[5] A vast historical literature exists on medieval Europe and specifically on the emergence and key features of feudalism. Historians remain conflicted over whether terms such as feudalism are useful. Specifically, the contractual aspects of feudalism emphasized by Ganshof (1951) and Bloch (1961, 1964) has been criticized by Brown (1974) and Reynonds (1994). From our perspective, the term feudalism describes a society where military power is decentralized among competing lords but in which there was also a recognized sovereign (who acts as "proposer" in our model).

[6] For Chaney and Blaydes (2013), the rise of feudalism has implications for the divergence between Europe and the Middle East. In contrast to Western Europe, Islamic states came to rely on slave soldiers. Landlords were alienated from political power as a consequence. Levels of political stability in these two regions of the world thus diverged centuries prior to the divergence in per capita income (Blaydes, 2017).

Lastly, we contribute to a nascent literature on the role of appropriability of resources in historical state formation. Mayshar, Moav and Neeman (2017) formalize the intuition that the ability of the ruler to appropriate revenue from agricultural lands was determined by dominant crop types available to farmers. Where output is more transparent, centralized states form earlier. Relatedly, Mayshar, Moav and Pascali (2022) build on the work of Scott (2017), and argue that the presence of agricultural crops whose output is highly observable and hence appropriable, such as wheat, leads to the formation of stronger states. Huning and Wahl (2023) develop this argument in the context of state formation in late medieval and early modern Europe. They find that more observable (and hence appropriable) agricultural output is associated with larger states. Such findings, however, only apply to fiscal states. We argue that the logic was quite different for feudal polities in which rulers sought to rule through their barons rather than by directly taxing the population.

2 Feudal coalitions in medieval Europe

This section describes the key features of the feudal world, features that we seek to capture in our model.

The term feudal is often applied to the entire medieval period (500 AD-1500 AD). By the feudal era or age of classic feudalism, however, historians typically refer to the period between 900–1200 (Bloch, 1961). This was the period when state structures were weak and military capacities were decentralized; large swathes of territories were governed by alliances or coalitions among military elites, forged through either war or peaceful means.

We use the term feudal, therefore, to refer to governance structures that comprised of alliances forged by mutual legal and military obligations but which were also hierarchical, e.g., there was a king and that king could allocate the resources of the realm, both productive and military; but the lords had their own military forces and hence the power to fight and rebel against the king.

A key characteristic of this coalition-based power structure was that it was precarious and often unstable — elites could move in and out of the ruling coalition. To retain power, a ruler had to maintain a coalition of the major landlords within her territory. This coalition could be continuously changing and a ruler had to be prepared to use violence to maintain her coalition. Political order in this environment rested not on formal institutions but on coalitions between individuals who could mobilize violence. The one long-lasting institution in this period was the Church (see Grzymala-Busse, 2020). But in every other respect power was not institutionalized but personal.

These characteristics were the product of European history and they distinguish medieval European polities from other parts of Eurasia. Following the fall of the Western Roman empire in the 5th century AD, Europe fragmented into many separate kingdoms (Scheidel, 2019). Whereas the Roman empire had possessed both a professional army and bureaucracy funded by a centralized fiscal system, its successor kingdoms lacked both of these crucial features. This transformation was complete by 600 (Wickham, 2005, 2009).

In the wake of this transformation, military power was decentralized. The core military resources of the successor kingdoms comprised the personal retinue or *comitatus* of the king. Major landowners formed similar bands of armed retainers (Young, 2018). In a world of decentralized violence capabilities, larger polities only formed when the ruler was successful in maintaining the loyalty of these landowners.

We motivate our analysis by considering two feudal polities: early medieval France and Norman England.

2.1 Early medieval France

By the late 5th century, Roman power had disintegrated in Northern Gaul. In its place, various warlords, Gallo-Roman aristocrats and Roman generals had established their own petty kingdoms.[7] Among these peoples were the Franks, and a particular subtribe, the Salian Franks based in modern Belgium.

Clovis became the leader of the Salian Franks in 482. Beginning with a small number of followers, Clovis sequentially united the various Frankish tribes, and through conquest or alliances consolidated his control over almost the entirety of Roman Gaul. But the coalition he built was transient. His successors controlled smaller territories and over time, political authority was increasingly localized. These centrifugal tendencies were arrested by the rise to power in Francia of the Carolingian dynasty. This period saw major attempts to restore centralized political authority (in addition to territorial expansion) (Collins, 1998; McKitterick, 2008; Wickham, 2009). But it was also relatively short-lived. External threats and internal conflict resulted in the breakdown of political order by the 9th century. The following period saw further decentralization, a period labeled by some as "the feudal revolution" (see Bisson, 1997).[8] In the kingdom of the Franks, the authority of the king was restricted to a small area around Paris and local lords entrenched their power (Bisson, 2009).

The resulting political order was one in which authority was local and personal. Centralized power fell to a low ebb. Duby (1981) emphasized the privatization of justice. Strayer (1970) writes of the absence of the state. Bisson (2009, 27) comments that "[r]oyal order was seldom centralized order". For Hintze (1975, 192), feudal polities were not states because their rulers "lacked the attributes of sovereignty—that is, independence beyond its borders and exclusive rights within them". Instead power rested on coalitions. Local lords fought, made peace, married, allied with one another, before falling out and fighting again. Describing 11th century Normandy, Barlow (2000, 6–7) notes that "This bald account of the rise and fall of a feudal principality suppresses the incessant, and to us bewildering, diplomacy and military campaigns which were necessary for its continuing existence. Each ruler competed with the others to construct a superior network of alliances. Princes

[7] These included Aegidius and his son Syargius at Soissons, Arbogast at Gaul; Britons fleeing Irish and Saxon invasions had settled in Armorica—what is now Brittany; and several different Germanic peoples occupied other territories (see Dam 2005) See also Wallace-Hadrill (1982, pp. 159–160); James (1982, pp. 26–28).

[8] Scholars debate the timing of the feudal revolution and whether the experience of northern Francia generalizes to other parts of Europe. These issues, while important, are not relevant to our analysis.

sought for patrons among the greater powers …They had also to make or threaten war against rebel and rivals: there was no court to which they could effectively appeal for the protection of their property". Violence or the threat of violence was endemic.[9]

2.2 Norman England

England after 1066 was a more consolidated realm than France. Nonetheless, though the kings of England were comparatively powerful, their power rested on their ability to maintain their coalition of lords, each of whom possessed their own lands, castles, and military resources.

Following the Norman Conquest, William the Conqueror (r. 1066–1087) made himself the ultimate lord of all land in the country which was held in fief from him. The Anglo-Saxon nobility lost their land and were replaced with men who had served with William.

This structure would characterize England's political economy for the remainder of the Middle Ages. The king was the most powerful landowner in the country and as feudal overlord, he possessed numerous other rights. But he had no standing army: beyond his own household knights, he relied on the armed forces of his lords.[10]

The king's ability to govern rested on his nobility. These nobles "did not represent sectors of society but pursued their own interests and those of their followers' …Politics was personal, not structural" (Bartlett, 2000, 28). William retained the ability to expropriate or redistribute the land of any of his lords (feudal tenure was not yet secure). There was no rule of law even for elites (North, Wallis, and Weingast 2009). Nor was there a codified rules of succession or an institution like a parliament to act as a coordination device. On his death, William I passed England to his second son William II "Rufus" (r. 1087–1100). Large-scale rebellions greeted Rufus on his ascension. He defeated the rebellious lords and a prospective invasion from Normandy threatened by his brother. But his rule remained insecure.

In all respects, therefore, Norman England remained a "fragile natural state" governed by a fairly loose coalition. Kings governed by making bargains with the most important and powerful lords who had to be coopted through the promise of land and resources.

There was civil war between 1139–1154. Major baronial rebellions reoccurred in 1172, 1215, and 1258–1265. Violent rebellions by dissatisfied lords continued to be the major source of political instability until the Tudors consolidated power in the 16th century by effectively outlawing private armies (see Greif and Rubin, 2024). Figure 1 records every year in which there was a battle or significant armed conflict within England due to either civil war or minor rebellions. By our estimation, there was at least one significant armed conflict in 14 % of the time between 1066 and 1500. If

[9] As Bloch (1964, p.134–135) observed: "The ever-present threat was one which lay heavy on each individual …War, murder, the abuse of power-these have cast their shadow over almost every page of our analysis …violence became the distinguishing mark of an epoch and a social system".

[10] Note that taxation did not play an important role in Norman England. The right to levy taxes had been established in Anglo-Saxon England as a means of providing defense against Viking attacks but it was allowed to lapse by Henry II in the 12th century.

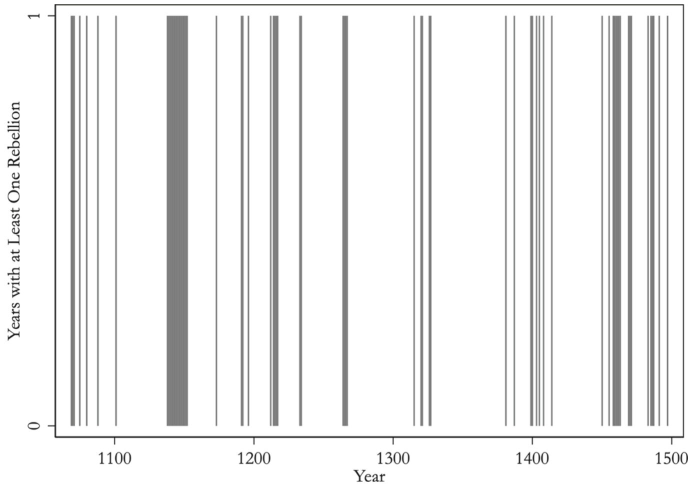

Fig. 1 Years during which there was at least one major violent (political) rebellion in England. Data was constructed based on information in (Allmand 1992, Prestwich 1997, Barlow 1999, Bartlett 2000, Hollister 2001, Carpenter 2003, Rubin 2005, Phillips 2011). We exclude rebellions and wars in Wales, Scotland or in France

we also include other moments of political conflict and crisis including the purging of major lords or political conflict that did not result in a battle, this increases to 18% of the time. If we exclude the periods when England was at war in France or Scotland, these proportions increase to 20% and 25% respectively.

Overall, there is strong support for Bloch's observation that "…even among the sovereigns whose power is most vaunted by the chroniclers, it would be impossible to find one who did not have to spend long years in suppressing rebellions …A petty rebel count entrenches himself in his lair and lo! the Emperor Henry II is held up for three months" Bloch (1964, p.133).

A notable feature of feudal rebellions was that failure was not fatal. Ranulf de Gernon (1099–1153), 4th Earl of Chester was originally loyal to King Stephen (r. 1135–1154). He rebelled when Stephen distributed some of his lands to the king of Scotland in response to an invasion. He later made peace with the King before rebelling again. Despite all of this, the King never attempted to remove Ranulf entirely by either killing him or taking all of his lands from him. This reflected a general rule. Pollack and Maitland (1895, 502) note that: "For two centuries after the Conquest, the frank, open rebellions of the great folk were treated with a clemency which, when we look back to it through intervening ages of blood, seems wonderful". Feudal law permitted lords the "withdrawal of loyalty, of the fides, from the king" (Ullman, 1961, 152). Thus "the vassal who resorted to war in defense of what he perceived as his rights could not be guilty of treason" (Cuttler, 1981, 5). This ensured that rebellious barons were typically permitted to rejoin the ruler's coalition at a future date.

3 Coalition formation with violent entry and exit

We now introduce a formal model of coalition formation with violent entry (conquest) and exit (rebellion).

A realm has a population of elites $N = \{i\}$ of size $|N| \geq 2$. Each elite i possesses resources $e_i \in \mathbb{R}^+$, which includes all productive resources that generate income, e.g. land, labor, capital, as well as military resources that provide protection, e.g. soldiers, weapons. As the medieval economy was Malthusian, we abstract away from the possibility of growth: e_i does not vary over time.

Now suppose there is a member of N who wants to be the ruler of the realm. She then proposes to every other member $i \in \{N - 1\}$ to join her coalition by committing her resources e_i to the ruler. In exchange, the member is promised a share in the coalition's total resources, which includes the ruler's own resources. The entry of $i \in \{N - 1\}$ into the coalition, however, may not be peaceful – the ruler can wage war against i to force her to join the coalition by conquest. Thus, any entry into the coalition may entail costs of fighting. Similarly, any member of the coalition can exit from the coalition by rebelling against the ruler, thereby incurring costs of rebellion. Coalition formation in a feudal environment can thus be depicted as a series of non-binding agreements between elites, which includes the possibility of violent entry and exit.

To be precise, at each time period t, N is partitioned into a coalition structure $\pi_t = (\omega, \{i_{\neg\omega}\})_t$, where ω is the coalition of the ruler (which includes the ruler), and $\{i_{\neg\omega}\}$ is a collection of singleton coalitions corresponding to each elite who is not in the ruler's coalition. This coalition structure determines the per-period payoffs of each $i \in N$ inasmuch as it affects the total resources of the ruler's coalition, as well as the share of those resources that is allocated to i. Specifically, an elite who is not a ruler but is in the ruler's coalition obtains payoff $\alpha_{i_\omega} E$ at t, where E is the total resources of the ruler's coalition at t – the sum of all individual resources committed to the ruler net of any costs of fighting or rebellion, and α_{i_ω} the share of those resources that is allocated to the elite at t. The ruler gets the remaining share $(1 - \sum \alpha_{i_\omega})$ and, thus, her payoff at t is $(1 - \sum \alpha_{i_\omega})E$. Lastly, the payoff at t of any member who is not in the ruler's coalition is $\hat{e}_{i_{\neg\omega}}$, which denotes her own resources net of any costs of fighting or rebellion. Thus, for each time period, the vector of payoffs for each elite (including the ruler) can be denoted as $\mathbf{u}_t = (\{\alpha_{i_\omega} E\}, (1 - \sum \alpha_{i_\omega})E, \{\hat{e}_{i_{\neg\omega}}\})_t$.

Describe the state s at time t as a pair of coalition structure and corresponding payoffs $s = (\pi, \mathbf{u})$. We can then show how, from an initial state $s_0 = (\{i\}, \{e_i\})$ in which each $i \in N$ is her own singleton coalition in control of her own resources, state s_0 evolves into other kinds of states. We are particularly interested in deriving conditions under which the state evolves into a consolidated realm, in which all $i \in N$ and their respective resources belong in the ruler's coalition, i.e. $s = \Big((\omega, \{i_{\neg\omega}\} = \{0\}), (\{\alpha_{i \in \{N-1\}} E\}, (1 - \sum \alpha_{i \in \{N-1\}})E\Big)$, or otherwise a fragmented realm in which some members remain independent, i.e. $s = \Big((\omega, \{i_{\neg\omega}\}), (\{\alpha_{i \in \omega} E\}, (1 - \sum \alpha_{i \in \omega})E, \{\hat{e}_{i_{\neg\omega}}\})\Big)$. The following game allows us to do so.

3.1 The feudal (bargaining) game

To motivate the game, consider a simple depiction of how a particular ruler, $j \in N$, might attempt to incorporate a particular elite $k \neq j$, $k \in \{N-1\}$, into her coalition, whether peacefully or by force.

Suppose j were to ask k to join j's existing coalition ω_{t-1} to form coalition $\omega_t = \{\omega_{t-1} \cup k\}$.[11] This would entail k committing her own resources to the ruler, in effect contributing e_k to the coalition's total resources. In exchange, j would promise k a share $\alpha_{k,t}$ of the total resources of the coalition (net of any costs) for each time period t that k would remain in the coalition. k could then either accept the proposal and be peacefully included in j's coalition, or reject it, in which case j would wage war against k to try to include k by conquest.

Before j would approach k, she would need to assemble a council from her existing coalition or, using the terminology in Ray (2007), an approval committee $A_t \subseteq \omega_{t-1}$, which is always non-empty and whose unanimous support would determine whether or not k could actually join the coalition.[12] In particular, should k refuse to join j's coalition, j would need the help of A_t to successfully conquer k — that is, a player i that is in A_t would have to incur fighting cost $c_i \in [0, e_i]$. The total fighting cost of the coalition that would be required to conquer k at t is thus $C_t = \sum c_i \mathbb{1}_{A_t}$, where $\mathbb{1}_{A_t}$ indicates membership in A_t. (If k accepted the offer, no fighting would ensue and k would join peacefully).

Any member of A_t who would disapprove of the entry of k would rebel against j and therefore exit from j's coalition. The rebel would seize back her own resources, but would incur rebellion cost $r_i \mathbb{1}_{R_t}$, where $\mathbb{1}_{R_t}$ indicates membership in the set of rebels $R_t \subseteq A_t$. Specifically, $r_i \in [0, e_i]$ is the portion of i's resources e_i that i cannot take back from the coalition once it has been committed to the ruler. It is therefore a measure of the extent to which e_i is appropriable by the ruler. The closer r_i is to zero, the less appropriable, or the more non-appropriable, e_i is by the ruler.

If any member of A_t rebelled, then j would be unable to cover fighting costs C_t and therefore unable to conquer k. Only if every member of A_t supported j would j be able to conquer k – each member of A_t is pivotal. The approval committee is thus a kind of minimal winning coalition *a la* Riker (1962) that is sufficient to incorporate k into the ruler's coalition.[13]

After forming A_t, j would then ask k to join her coalition. k would either accept or reject j's offer, in which case k would fight j in battle and incur cost of fighting $c_k \in [0, e_k]$.

This scenario describes a potential pairwise interaction between a ruler j and an elite k. Its outcome would affect the coalition structure π of N. For instance, if k would

[11] We specify set union, rather than $\omega_t = \{\omega_{t-1}, k\}$ to capture the possibility that k is already in ω_{t-1}, in which case j asks k to re-affirm her membership in j's coalition.

[12] The assumption that A_t is non-empty is easy to make as ω_{t-1} includes the ruler and therefore the ruler can be her own approval committee.

[13] The ex-post probability of k being conquered may be anything from 0 to 1. See subsequent game and results. In 1215 King John called a council to confront the rebellious barons led by Robert Fitzwalter and Eustace de Vesci. However, only 28 barons responded and as a result John was unable to defeat the rebels and came to terms (see Desierto, Hall, and Koyama, 2023).

end up in j's coalition, then the ruler's coalition ω would expand to $\{\omega_{t-1} \cup k\}$, while the singleton coalition of k would become a null set. If rebellion occurred, ω would contract to $\{\omega_{t-1} \setminus R_t\}$, while each of the rebels would become her own singleton coalition.

The outcome of the pairwise interaction would also affect the vector of payoffs $\mathbf{u}$. For one thing, the entry of k into, and the exit of R_t from, ω would affect the total resources E of the ruler's coalition. For another, it could affect the allocation of E among members of ω. This is because to accommodate the entry of k, some existing member of ω_{t-1} would have to have a smaller share (as the sum of shares is necessarily equal to one). On the other hand, the exit of rebels R_t would mean that the remaining members in $\{\omega_{t-1} \setminus R_t\}$ could have larger shares.[14]

Thus, the pairwise interaction between j and k would affect the state $s = (\pi, \mathbf{u})$. As the interaction is repeated between j and every other elite in N, the state would evolve.

We cast the above scenario into an infinitely-repeated pairwise bargaining game that generates a particular state $s = (\pi, \mathbf{u})$ at each time period. Before doing so, it is useful to add some notation. Partition the vector of payoffs into $\mathbf{u} = (u_j, \{u_{i \neq j}\})$, where u_j is the payoff of ruler j and $\{u_{i \neq j}\}$ the payoffs of every other elite $i \neq j$. Let $s^L = \left((\{\omega_{t-1} \cup k\}^L, \{i_{\neg\omega}\}^L), (u_j^L, \{u_{i \neq j}\}^L)\right)$ be a state in which a particular elite k joins j's coalition and j's particular approval committee is loyal, and denote as S^L the set of all such states. Similarly, let $s^R = \left((\{(\omega_{t-1} \setminus R_t) \cup k\}^R, \{i_{\neg\omega}\}^R), (u_j^R, \{u_{i \neq j}\}^R)\right)$ be a state in which k joins j's coalition but members $R_t \subseteq A_t$ of a particular committee rebel, and denote as S^R the set of all such states. One can then denote as L the state in S^L that gives the highest payoff to j. That is, $L = s^L$ such that $\forall (s^L, s^{L'}) \in S^L$, $u_j^L > u_j^{L'}$. Similarly, we denote as R the state in S^R that gives the highest payoff to j, i.e. $R = s^R$ such that $\forall (s^R, s^{R'}) \in S^R$, $u_j^R > u_j^{R'}$. This allows us to limit the number of pure actions of j in the following game to two – L and R.

Thus, starting from initial state $s_0 = (\{i\}, \{e_i\})$ in which each $i \in N$ is her own singleton coalition, let the following sequence of events occur at each subsequent time period $t = 1, 2, \ldots, \infty$:

1. A pair of players (j, k) is randomly drawn from N, with j the proposer, and k the responder. More specifically, $(j, k) = (a, k)$ where k is randomly drawn (with replacement) from $N \setminus a$ and a is randomly drawn (with replacement) from N and thereafter fixed until all members of $N \setminus a$ have been drawn to play at least once.[15]
2. j chooses between any of the two states in which k is included in j's coalition: L or R, and proposes this to k.
3. k chooses between accepting (A) or rejecting the proposal, which implies fighting (F) against j.

[14] This is why it might be incentive compatible for a ruler to knowingly include a would-be rebel in her approval committee.

[15] Since a and k are randomly drawn with replacement, the time horizon of any proposer and any responder is in effect infinite.

4. A move to a new state occurs, depending on the chosen actions of j and k. Denote as $s_{12} = (\pi_{12}, \mathbf{u}_{12})$ the state when j chooses $1 = \{L, R\}$ and k chooses $2 = \{A, F\}$. Then there are four (pure) states that can be implemented: $s_{LA}, s_{LF}, s_{RA}, s_{RF}$.

Some remarks are in order.

3.1.1 The proposer

Step 1 implies that although the proposer is randomly selected, only one proposer is drawn at a time. That is, only when all other players are drawn to respond at least once will there randomly 'appear' another proposer. We make this specification to match our empirical setting. It was Clovis, for instance, who sought to consolidate Post-Roman Gaul by forging alliances with, or conquering, previously independent Frankish tribes. England following the Norman Conquest, where prospective rulers generally appeared one at a time, was what historians call a consolidated feudal monarchy (Painter, 1951).

There are, of course, some notable exceptions in which more than one elite had seemingly equally legitimate claims to the royal throne. When Henry I (r. 1100–1135) died in 1135, the throne was fought over by his daughter Matilda and nephew Stephen (r. 1135–1154), which plunged England into civil war. Similarly, the Wars of the Roses (1455–1487) ensued after Richard of York challenged Lancastrian rule by attempting to claim the throne from Henry VI (r. 1422–1461). As these two examples show, feudal coalition formation with simultaneous proposers can lead to a different kind of violence – civil war, which makes it both important and distinct so as to warrant a separate model. We leave this avenue open for future research.

3.1.2 The proposal

In step 2, we abstract from the exact process by which approval committees are formed. j can effectively form any committee, which may be subject to particular constraints. For instance, it may not always be possible to include in the approval committee only members with the least costs of fighting, or those with the highest costs of rebellion. The ruler, instead, may be constrained to always include her family members in the approval committee. Such constraints can limit the number of approval committees that j can form. Among this set, some committees may be completely loyal, while others may include some would-be rebels.

Step 2 simplifies a number of simultaneous considerations that j makes before proposing to k. One is the list of committees that she can form (given any constraints), another is whether she prefers a loyal approval committee or is willing to suffer some rebellion (which would affect the total net resources of his coalition) and, lastly, the shares to be allocated to each remaining member of the coalition. The key simplification underlying step 2 is that j would never want to offer to k a state s if she could offer another state s' that would give her a higher payoff. Thus, by constructing L as the state in S^L that gives her the highest payoff under loyalty, and R as the state in S^R that gives her the highest payoff under rebellion, we are able to restrict the choice of j between, from her perspective, an 'optimal' state of loyalty and an 'optimal' state of rebellion.

We can further characterize L and R to suit our setting. To do this, we first make the following behavioral assumption pertaining to feudal elites.

Assumption 1 There exists, for each $i \in N$, a 'reservation' share $\underline{\alpha}_i$, below which i would always prefer to be independent. The sum of reservation shares of the elites of a realm is at most one, i.e. $\sum_i^N \underline{\alpha}_i \leq 1$.

Assumption 1 implies two things. One, that i would never desire to be in a (non-singleton) coalition ruled by any other elite in the realm unless she is guaranteed a share of at least $\underline{\alpha}_i$ in that coalition. This is irrespective of the size of the coalition's resources. We can think of $\underline{\alpha}_i$ as capturing i's desire for independence, which is distinct from her desire for resources. An elite who only considers joining a coalition of which she gets at least fifty percent of the pie desires more independence than another who considers joining when she gets at least ten percent, irrespective of the size of the pie. One possible reason for why an elite might refuse to join rich coalitions is to preserve one's national pride or cultural identity. Feudal history is replete with examples. A notable one is the continued refusal of Welsh princes, from Llywelyn the Great to Llywelyn ap Gruffydd, to subject themselves to English rule despite relatively generous offers (at least in material terms i.e. land and titles in England) by English kings, until Edward I finally conquered Wales in 1277–1283 (Prestwich, 1997, 191).

The other implication of Assumption 1 is that what constitutes a realm N are a set of elites (and their resources) whose sum of reservation values cannot be larger than one. This sets a boundary as to who and how many elites can form one consolidated unit. This can explain why too many independent elites, even if they are geographically proximate to each other and even related to each other by blood or marriage, remain separated into distinct countries.[16] If neighboring realms already have fixed and distinct national/cultural identities, e.g. France (F) and England (E) in the late 15th and 16th centuries, then it is unlikely that an elite residing in F is in the set N_E of elites playing the feudal game to consolidate England, because her reservation share, should she play that game, would be very high, if not equal to one. Indeed, it is inconceivable during these periods that a French lord would even consider serving an English king.[17]

Since, given a particular approval committee and composition of her coalition, j can always increase her payoff by lowering the shares of her coalition members to their reservation values, L and R are offers in which each coalition member (other than j) is allocated her reservation share at each time period she remains therein. Moreover, it can be shown that this is also true in each of the pure states $s_{LA}, s_{LF}, s_{RA}, s_{RF}$ that can occur after each pairwise play. Thus:

[16] This is not to say that geography is not also important in determining border formation (see Abramson, 2017; Kitamura and Lagerlöf, 2019; Ferández-Villaverde et al., 2023) Our model simply accommodates cases in which there are no obvious geographical limitations to consolidation, and yet elites maintain separate boundaries to preserve their identity, nationhood, or culture, provided that such identities are already fixed and do not, for instance, depend on who the proposer is.

[17] In the 11th and 12th centuries, however, French and English identities were still in flux, and likely varied with the identity of the proposer. After 1066, the feudal elite of England spoke French and saw themselves as Norman French. Had the English Harold Godwinson remained King instead of being deposed by William of Normandy, the French-speaking elites might not have wanted to "join" England.

Lemma 1 *At each* $t = \{1, 2, ..., \infty\}$, *the state is such that* $i_\omega \in \omega_t$ *is allocated her reservation share* $\underline{\alpha}_{i_\omega}$ *at each time period she remains in* ω. *Meanwhile,* j *obtains share* $(1 - \sum \underline{\alpha}_{i_\omega}) \geq \underline{\alpha}_j$ *at all* t.

Proof All proofs are in the Appendix.

This result further simplifies the process underlying step 2. By fixing the allocation of shares at all time periods among members of j's coalition to their reservation shares, the construction of the payoffs only requires total net resources E. In turn, E is easily deduced from the players' actions since these actions determine the inclusion or exclusion of k into the coalition, and any exit of rebels and, therefore, E.

3.1.3 States

Step 4 specifies four pure states that can be implemented, depending on the outcome of a pairwise play: $s_{LA}, s_{LF}, s_{RA}, s_{RF}$.

State s_{LA} is implemented after j proposes to k a state in which k is in j's coalition and j's approval committee is loyal, and k accepts. Thus, under state s_{LA}, k has peacefully joined j's coalition, contributing e_k to it, and obtaining share $\alpha_k = \underline{\alpha}_k$ (by Lemma 1) of the resources of the coalition at each time t that she remains therein.

State s_{LF} is implemented after j proposes to k a state in which k is included in j's coalition and j's approval committee is loyal, but k rejects the proposal, and so fighting ensues. With a loyal committee, j successfully conquers k, albeit after incurring fighting cost C. Thus, under s_{LF}, k has incurred fighting cost c_k and has been conquered. She has therefore surrendered her resources (net of c_k) to the coalition and obtains fraction $\alpha_k = \underline{\alpha}_k$ of the total resources of the coalition (net of C) at each time t that she remains therein.

State s_{RA} is implemented after j proposes to k a state in which k is in j's coalition and some or all of j's approval committee are in rebellion, and k accepts. No fighting ensues, but the rebels exit from the coalition. Thus, under state s_{RA}, k has peacefully joined j's coalition, contributing e_k therein, and obtaining share $\alpha_k = \underline{\alpha}_k$ of the coalition's resources at each time t that she remains therein. The coalition's resources have been reduced by the sum of the individual resources of the rebels, net of the rebellion costs they incurred at the time of their exit. (Each rebel takes back her own resources, except what is appropriable by the ruler).

Finally, state s_{RF} is implemented when j proposes to k a state in which k is in j's coalition and some or all of j's approval committee are in rebellion, but k rejects the proposal, and fighting ensues. j incurs fighting cost C but is unable to conquer k because of the rebellion. Thus, under state s_{RF}, k has incurred fighting cost c_k and remains independent. She has no share in the coalition's resources, which have been reduced by coalition fighting costs and the resources that the rebels have taken back. Instead, k keeps all of her own resources (net of c_k).

3.1.4 Strategy profiles

The players of the feudal bargaining game are thus $\{i\} = N$, where i can be a proposer j or responder k at any time period t characterized by a particular state s. To construct a strategy profile, we specify player i's actions as a proposer and as a responder given s. As proposer, $i = j$ chooses between L and R. Let $\mu_j(s)$ denote the probability that the proposer chooses L given s. As responder, $i = k$ chooses between A and F. Let $\lambda_k(s)$ be the probability that a responder chooses A given s. A strategy profile $\sigma = \{(\mu_j, \lambda_k)\}_i$ is a collection of pairs of proposer-responder actions over all i, which is defined for s. This induces the following expected payoffs for each player $i = \{j, k\}$ drawn to play when the state is s:

$$\begin{aligned} V^k(\mu_j, \lambda_k = 1, s) &= (1-\delta)u^k(s) \\ &+\delta\Big[\mu_j V^k(\mu_j = 1, \lambda_k = 1, s_{LA}) + (1-\mu_j)V^k(\mu_j = 0, \lambda_k = 1, s_{RA})\Big] \end{aligned} \tag{1}$$

$$\begin{aligned} V^k(\mu_j, \lambda_k = 0, s) &= (1-\delta)u^k(s) \\ &+\delta\Big[\mu_j V^k(\mu_j = 1, \lambda_k = 0, s_{LF}) + (1-\mu_j)V^k(\mu_j = 0, \lambda_k = 0, s_{RF})\Big] \end{aligned} \tag{2}$$

$$\begin{aligned} V^j(\mu_j, \lambda_k, s) &= (1-\delta)u^j(s) \\ &+\delta\Big[\mu_j\Big(\lambda_k V^j(\mu_j = 1, \lambda_k = 1, s_{LA}) + (1-\lambda_k)V^j(\mu_j = 1, \lambda_k = 0, s_{LF})\Big) \\ &+(1-\mu_j)\Big(\lambda_k V^j(\mu_j = 0, \lambda_k = 1, s_{RA}) + (1-\lambda_k)V^j(\mu_j = 0, \lambda_k = 0, s_{RF})\Big)\Big], \end{aligned} \tag{3}$$

where δ is the discount factor and $u^k(s)$, $u^j(s)$ denote the one-period payoffs given s.

The feudal bargaining game is akin to Ray's (2007) proposal-based model of coalition formation in which there is a finite set of players, a compact set of states, an infinite time horizon, an initial state, a protocol describing the proposer and order of respondents at each time period, subsets of players that can approve the move from each state to another, and for each player, a continuous one-period payoff function and discount factor common across each players. However, two things are notably different. One is that we give the proposer the option to deliberately choose an approval committee that does not approve the proposal. The other is that we allow violence to occur during entry into (conquest), and exit from (rebellion), the coalition.

3.2 The feudal political economy (FPE) equilibrium

We now define equilibria in the feudal game. To do so, we first define a particular type of pair of proposer-responder actions for $i = \{j, k\}$.

Definition 1 The pair (μ_j, λ_k) of proposer-responder actions for $i = \{j, k\}$ is an **optimal action pair** if: $\lambda_k = 1$ if $V^k(\mu_j, \lambda_k = 1, s) > V^k(\mu_j, \lambda_k = 0, s)$, equals 0 if the opposite inequality holds, and lies in [0, 1] if equality holds; $\mu_j = \arg\max V^j(\mu_j, \lambda_k, s)$.[18]

[18] Note, then, that if $\lambda_k = 1$, $\mu_j = 1$ maximizes $V^j(\cdot)$ if $V^j(\mu_j = 1, \lambda_k = 1, s_{LA}) > V^j(\mu_j = 0, \lambda_k = 1, s_{RA})$, 0 if the opposite inequality holds, and lies in [0, 1] if equality holds. Analogously, if $\lambda_k = 0$, $\mu_j = 0$ maximizes $V^j(\cdot)$ if $V^j(\mu_j = 1, \lambda_k = 0, s_{LF}) > V^j(\mu_j = 0, \lambda_k = 0, s_{RF})$, 0 if the opposite inequality holds, and lies in [0, 1] if equality holds. Note that if λ_k lies in [0, 1], the value of μ_j that maximizes $V^j(\cdot)$ may be 1 or 0, or may lie in [0, 1]. For instance, if $V^j(\mu_j = 1, \lambda_k = 1, s_{LA}) > V^j(\mu_j = 0, \lambda_k = 1, s_{RA})$ and $V^j(\mu_j = 1, \lambda_k = 0, s_{LF}) > V^j(\mu_j = 0, \lambda_k = 0, s_{RF}$, then $\mu_j = 1$ maximizes $V^j(\cdot)$.

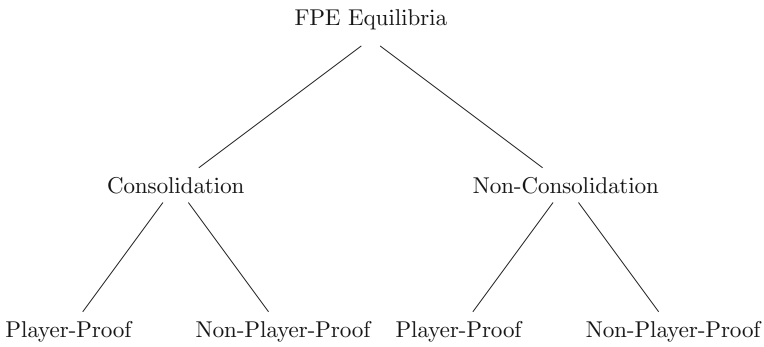

Fig. 2 Equilibria of the feudal bargaining game

One can then define equilibria in terms of optimal action pairs:

Definition 2 A strategy profile $\sigma = \{(\mu_j, \lambda_k)\}_i$ is a **Feudal Political Economy (FPE) equilibrium** if for each $i = \{j, k\}$, (μ_j, λ_k) is an optimal action pair.

One can also refine the FPE equilibrium using optimal action pairs. For reasons that will be obvious in Sect. 4 – when we derive conditions under which alliances are made and the realm is consolidated, one can consider FPE equilibria in which there is only one optimal action pair for each player. That is, all players are associated with the same optimal action pair, such that the equilibrium, in this specific sense, is "player-proof".

Definition 3 An FPE equilibrium is **player-proof** if the optimal action pair (μ_j, λ_k) for $i = \{j, k\}$ is the same for all i.

While restrictive, player-proof equilibria can serve as benchmark – as we show in Sect. 4, they can approximately describe the type of polity that is generated by the feudal game.

Lastly, we define a particular type of FPE equilibrium in which all players, when playing as proposer, choose $\mu_j = 1$. In this equilibrium, no rebellion can occur, which means all respondents join the coalition, whether peacefully or by conquest. Because this equilibrium is characterized by full entry into, and no exit of players from, j's coalition, it gives rise to a consolidated realm.

Definition 4 An FPE equilibrium is a **consolidation equilibrium** if the optimal action pair for all $i = \{j, k\}$ is $(\mu_j = 1, \lambda_k)$.

It follows that if for some i, the optimal action pair is $(\mu_j \neq 1, \lambda_k)$, then the equilibrium is not a consolidation equilibrium. Thus, the set of all FPE equilibria consists of the set of consolidation, and the set of non-consolidation, equilibria. Any player-proof equilibrium is either a consolidation or a non-consolidation equilibrium (Fig. 2).

4 Alliances and consolidation

We now apply the equilibrium concepts in the previous section to answer questions of interest about the feudal world. First, under what conditions does a responder k ally with proposer j (by joining j's coalition), and is the alliance peaceful or achieved through violent conquest? Second, what determines the likelihood that all responders remain allied to j in a single coalition – that is, whether a realm is consolidated or remains fragmented?

The key variables are resources, the extent to which these resources are non-appropriable by the ruler, the costs of fighting of approval committees and of responders.

Theorem 1 establishes that a responder k is more likely to ally with j when the resources of j and all other responders – actual and potential coalition members, are large and appropriable, as this means that the coalition's total resources (those that remain after any rebellion) are large and therefore the alliance is valuable to k. It deters k from fighting and induces her to accept the proposal peacefully. If resources are appropriable, the alliance is formed even if some members of j's approval committee rebel. In this case, j would be more likely to let the rebellion occur since rebels cannot take much away from the coalition, but since k is likely to join peacefully, the alliance between j and k is formed, in spite of any rebellion from other members.

Theorem 1 also shows that an alliance is more likely to occur when j's approval committee is good at fighting. In this case, they are likely to deter k from fighting. In addition, it makes j less likely to want them to rebel and exit the coalition. Thus, it is likely that a loyal approval committee has low fighting costs, which means that the alliance between k and j is likely to be made, whether peacefully or by conquest.

The fighting cost of the responder has an ambiguous effect on the probability of alliance. On the one hand, a responder that is weak is easy to conquer, which could even deter the responder from fighting. Thus, whether peacefully or by conquest, a weak responder is likely to end up in j's coalition. On the other hand, a weak responder is also not very valuable in the coalition, which obviates the need for j to have a loyal committee in order to conquer the responder. The exit of the rebels and the exclusion of k would increase j's share in the total net resources of the coalition, which would make her more likely to allow rebellion. Thus, a weak responder may avoid getting conquered and may be more likely to remain outside the coalition.

Taking into account all pairwise interactions and possible alliances between any j and any k, we then analyze the likelihood of a consolidation equilibrium, in which each $i \in N$ remains in a single, grand, coalition. The same variables determine the likelihood of such consolidation, albeit in a different way. This is because the variables affect *every* entry into, and *every* exit from, the ruler's coalition.

Theorem 2 establishes that large resources make consolidation more likely – they tend to attract members into the coalition, and keep them there. However, the more appropriable these resources are, the *less* likely is consolidation. Appropriable resources make joining the coalition attractive, but they also make rebellion easier

(Theorem 1). In this case, j is more likely to allow rebellion since it is less harmful to j – rebels can only retrieve a small amount of resources, and can easily be enticed back into the coalition because appropriable resources make the coalition attractive to outsiders.

Theorem 2 (and Corollary 1) also show that costs of fighting affect the likelihood of consolidation, but the costs that matter are those of the weakest responder, and of the weakest approval committee among all committees formed. The smaller these costs, the more likely is consolidation. If the weakest approval committee is sufficiently strong, then j will conquer every responder, and everyone eventually joins the coalition. If the worst responder is sufficiently strong such that she is worth keeping in the coalition, then everyone else is valuable and it is always worth preventing rebellion.

With the exception of the size of resources, which always makes joining and remaining in the coalition attractive, and therefore increases the probability of alliance formation and consolidation, the effect of the other variables are more nuanced and cannot be readily deduced. For this reason, we show the logical progression towards Theorem 1 and Theorem 2.

4.1 Pairwise alliance

We derive conditions under which alliances are made and describe the nature of the alliances made in equilibrium. We first obtain the following result.

Proposition 1 ***Pairwise Outcomes*** *In equilibrium, the outcome from any pairwise play can be any of the following:*

1. ***Peaceful Alliance*** *between j and k, i.e.* $[\mu_j = 1, \lambda_k = 1]$.
2. ***Alliance by Conquest*** *of k by j, i.e.* $[\mu_j = 1, \lambda_k \in [0, 1)]$.
3. ***Alliance with Unrest*** *in which k accepts to join j's coalition, but some members rebel i.e.* $[\mu_j \in [0, 1), \lambda_k = 1]$.
4. ***No Alliance*** *between j and k, i.e.* $[\mu_j = 0, \lambda_k = 0]$.

It can then be shown that the likelihood of obtaining each outcome depends on a key set of variables.

Theorem 1 ***Determinants of Pairwise Outcomes*** *The following variables determine the likelihood of each type of pairwise outcome from any pairwise play in equilibrium:* $e_j, \{e_k\}, r_j, \{r_k\}, c_k, \{C_t\}$.[19] *The directions of the effect of each variable are summarized in the table below:*

[19] Note that e_j is the resources of j, $\{e_k\}$ the collection of resources of every responder, r_j a measure of appropriability of j's resources, $\{r_k\}$ the collection of the respective measures of appropriability of the resources of every responder, c_k the cost of fighting of the currently-drawn responder, and $\{C_t\}$ the collection of fighting costs of every approval committee formed.

	Peaceful alliance $[\mu_j = 1, \lambda_k = 1]$	Alliance by conquest $[\mu_j = 1, \lambda_k \in [0, 1)]$	Alliance with unrest $[\mu_j \in [0, 1), \lambda_k = 1]$	No alliance $[\mu_j = 0, \lambda_k = 0]$
e_j	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\{e_k\}$	$\uparrow$	$\downarrow$	$\uparrow / \downarrow$	$\downarrow$
r_j	$\uparrow$	$\uparrow / \downarrow$	$\uparrow / \downarrow$	$\downarrow$
$\{r_k\}$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
c_k	$\uparrow / \downarrow$	$\downarrow$	$\uparrow$	$\uparrow / \downarrow$
$\{C_t\}$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$

Theorem 1 establishes that the determinants of the outcome of any pairwise play are the resources of the players, how appropriable these resources are, and the players' costs of fighting. Specifically:

4.1.1 Resources of j, e_j, and of each k, $\{e_k\}$

Theorem 1 shows that the proposer's own resources, e_j, increase the likelihood of peaceful alliance and of alliance with unrest, and decreases that of alliance by conquest and of no alliance. Meanwhile, the resources of each of the responders, $\{e_k\}$, increase the likelihood of peaceful alliance, decrease that of alliance by conquest and of no alliance, and have an ambiguous effect on the likelihood of alliance with unrest.

4.1.2 Appropriability of j's resources, r_j, and of each of k's, $\{r_k\}$

Theorem 1 also shows that the extent of appropriability of the proposer's resources, r_j, increases the likelihood of peaceful alliance, decreases that of no alliance, and has ambiguous effects on the likelihood of alliance with unrest and of alliance by conquest. Meanwhile, the extent of appropriability of the resources of each of the responders, $\{r_k\}$, increase the likelihood of peaceful alliance and of alliance with unrest, and decrease that of alliance by conquest and of no alliance.

4.1.3 Cost of fighting of k, c_k, and costs of fighting of each approval committee formed, $\{C_t\}$

Finally, Theorem 1 shows that the cost of fighting of the responder, c_k, decreases the likelihood of alliance by conquest, increases that of alliance with unrest, and has ambiguous effects on the likelihood of peaceful alliance and of no alliance. Meanwhile, the costs of fighting of each approval committee formed, $\{C_t\}$, decreases the likelihood of peaceful alliance and of alliance by conquest, and increases that of alliance with unrest and of no alliance.

The intuition behind Theorem 1 is as follows. Large and appropriable resources make it more likely that accepting j's proposal is the dominant action for k, but makes rebellion more likely since it is less costly for j when rebels can only take back a small part of (large) total resources. This makes peaceful alliances and alliances with unrest (rebellion) more likely, and alliance by conquest – which requires zero rebellion, less likely. If the approval committee is strong – its fighting costs small, j is more likely to want to keep them loyal, which increases the likelihood of alliances by conquest and decreases alliance with unrest.[20] When responder k has low fighting costs, she may be more likely to fight, which may decrease the probability of a peaceful alliance. A strong responder also induces j to keep the approval committee loyal, thereby decreasing the likelihood of alliance with unrest and increasing that of alliance by conquest.

4.2 Consolidation (no rebellion)

Our other main result concerns the likelihood of consolidation. Specifically, we establish conditions under which a consolidation equilibrium is obtained, that is a realm in which none of the elites rebel. Since some consolidation equilibria are player-proof, it is useful to derive these more restrictive type of equilibria. They serve as benchmark equilibria that can approximate empirical patterns observed in feudal polities. Proposition 2 shows that there only four types of player-proof equilibria.

Proposition 2 ***Player-proof equilibria***

There exist only four types of player-proof equilibria:

1. ***Peaceful Consolidation**, in which the optimal action pair for all $i = \{j, k\}$ is $(\mu_j = 1, \lambda_k = 1)$.*
2. ***Consolidation by Conquest**, in which the optimal action pair for all $i = \{j, k\}$ is $(\mu_j = 1, \lambda_k \in [0, 1))$.*
3. ***Fragmented Polity**, in which the optimal action pair for all $i = \{j, k\}$ is $(\mu_j \in [0, 1), \lambda_k = 1)$.*
4. ***Independent Territories**, in which the optimal action pair for all $i = j, k\}$ is $(\mu_j = 0, \lambda_k = 0)$.*

Lemma 3 in the Appendix shows that each of these player-proof equilibria are obtained whenever a set of conditions holds for every $i = \{j, k\}$. Lemma 3 specifies four such sets of conditions. When the first set holds for all $i = \{j, k\}$, then all proposers choose $\mu_j = 1$ and all responders choose $\lambda_k = 1$. We call this player-proof equilibrium as one of peaceful consolidation, since all proposals are accepted without going to battle, and no one rebels from a single, grand coalition. When the second set of conditions holds for all $i = \{j, k\}$, then all proposers choose $\mu_j = 1$ and all responders choose $\lambda_k \in [0, 1)$. In this case, there is always some probability of fighting, but j's coalition always wins since approval committees are always loyal. We call this player-proof equilibrium consolidation by conquest. When the third set of conditions holds for all $i = \{j, k\}$, then all proposers choose $\mu_j \in [0, 1)$ and all responders choose $\lambda_k = 1$. Every responder (peacefully) joins the coalition, but because there is always some probability of rebellion, this player-proof equilibrium describes a fragmented polity. Lastly, when the fourth set of conditions holds for all $i = \{j, k\}$, then all proposers choose $\mu_j = 0$ and all responders choose $\lambda_k = 0$. No alliance is ever made, and each player remains its own singleton coalition. In other words, this player-proof equilibrium describes independent territories.

[20] A strong approval committee can also deter k from fighting which increases the likelihood of peaceful alliance.

Note, then, that two types of player-proof equilibria — peaceful consolidation and consolidation by conquest, describe a consolidated realm. There are, of course, many non-player proof equilibria, as there is no reason why the same set of conditions in Lemma 3 should hold for all $i = \{j, k\}$. Some of these non-player proof equilibria can also give rise to a consolidated realm. In particular, any equilibrium in which the optimal action pair is either $(\mu_j = 1, \lambda_k = 1)$ or $(\mu_j = 1, \lambda_k \in [0, 1))$ is one in which no rebellion ever occurs. Thus, even if a responder rejects the proposal, she still ends up in the coalition after being conquered. There is full entry into, and no exit from, the coalition.

Theorem 2 establishes conditions that give rise to a consolidated realm, whether the particular equilibrium is player-proof or non-player proof.

Theorem 2 ***Likelihood of Consolidation***

Denote as c_i^* the largest (individual) cost of fighting among all $i \in N$, and C_t^* the largest coalition fighting cost in $\{C_t\}$. Then the likelihood that a consolidation equilibrium is obtained increases with $\{e_i\}$ and decreases with c_i^*, C_t^*, and $\{r_i\}$.[21]

The intuition follows mostly from Theorem 1, to the extent that the variables affect each entry into, and each exit from, any j's coalition. Since we now consider the entire (infinite-horizon) game, and not just a single pairwise play, we can generalize to all players by considering, e.g. resources of all players, whether drawn as proposer or responder. Large individual resources, which increase the total coalition resources that can be allocated among the members, increase the likelihood of any entry and decrease the likelihood of any exit (rebellion). Thus, resources increase the likelihood of consolidation. If these resources are mostly appropriable by the ruler, however, rebellion becomes less costly to any j. Since any j is now more likely to put forth proposals to responders that can generate rebellion, there is less likelihood of consolidation. Similarly, large fighting costs decrease the likelihood of consolidation because while weak fighters are easy to conquer, making any entry more likely, they are also easy to let go from the coalition, making any exit more likely.

Note, however, that while the entire vector of player resources (and their respective appropriability) in the realm determines the likelihood of consolidation, in terms of costs of fighting, only the largest costs matter. That is, Theorem 2 implies the following.

Corollary 1 ***The Weakest Link***

To determine the likelihood of consolidation, one considers the worst, and not the best, members of the realm. In particular, the realm is ***likely to consolidate*** *if it is incentive compatible for the proposer to have* ***the worst fighter join, and the worst approval committee stay****, in the coalition.*

[21] Note that $\{e_i\}$ is the collection of resources of each i, which thus includes those of any proposer. Similarly, the collection of measures of appropriability $\{r_i\}$ include those of any proposer. Strictly speaking, the likelihood is decreasing in the resource-appropriability of responders, $\{r_k\}$, but the resource-appropriability of any proposer, r_j, has a non-monotonic effect. Specifically, there exist thresholds $r_{jj}^0 < r_{jj}^*$ such that the likelihood of consolidation is decreasing in $r_j \in [0, r_{jj}^0)$, increasing in $r_1 \in [r_{jj}^0, r_{jj}^*]$, and constant in $r_j \in (r_{jj}^*, \infty)$. (See the proof of Theorem 2 in the Appendix for details). That we establish that the likelihood is decreasing in $\{r_i\}$ implies that we consider, for any proposer j, only the region $r_j \in [0, r_{jj}^0)$.

The intuition is simple but powerful – when any ruler (proposer) is willing to share coalition resources to keep even the least (militarily) valuable members, then she would be willing to do so for everyone else. The result then implies that even if the rest of the members are just marginally better at fighting, consolidation is still likely. That is, not everyone in the coalition has to be a strong warrior.

In contrast, every player's resources contribute to the likelihood of consolidation in that they determine each player's gain from staying in j's coalition. Resources cumulate, and so every contribution is relevant. The non-appropriability of resources (low $\{r_i\}$) also cumulate in the sense that greater total non-appropriability makes it less likely for any ruler to allow rebellion. In equilibrium, exit is less likely.

One can therefore expect consolidated realms to have large, non-appropriable, total resources (large $\{e_i\}$, low $\{r_i\}$), in which the militarily weakest player and approval committee are sufficiently strong (low c_i^*, low C_t^*).

5 Discussion

We can now apply our model to the feudal world. The key variables are resources and the extent to which they are appropriable or non-appropriable and the distribution of fighting costs.

Resources include both economic and military resources. Economic resources in the feudal world were primarily agricultural – land quality and suitability for farming certain crops, were of fundamental importance. How much of these resources were non-appropriable by the ruler depends on many factors. Access to specific location fundamentals that acted as natural barriers, e.g. mountains, coasts or rivers, for instance, made it harder for the ruler to appropriate the land and other resources of the feudal lord. This, therefore, made the lord's resources more non-appropriable as they made it easier to guard the lord's land or transport output away from the ruler's reach should the lord exit the coalition.

Military resources include soldiers, horses, weapons, and fortifications that the ruler could use to attack outsiders, and defend insiders, of her coalition once committed to the coalition. However, some of these the ruler might not easily appropriate – soldiers, for instance, could be especially loyal to their immediate feudal overlord. Should the lord rebel, he could easily take his soldiers with him.

Unfortified or indefensible land can be thought of as highly appropriable by the ruler. In contrast, investments or technologies that enabled a lord to defend his possession, most notably his castle, were themselves difficult to appropriate by the ruler and, in addition, made his surrounding land and resources less appropriable and therefore easier to seize back from the ruler.

Lastly, fighting costs reflected the military ability of the elites – the lower the costs the higher the ability. Individual fighting prowess was a key variable in the medieval world. Someone who could win battles using a small amount of resources was more militarily able than someone who had to use more resources. Thus, military ability does not only refer to the physical fighting capability of the elite, but also reflects the military technology she uses. More effective technologies would lower the elite's cost of fighting.

How do these variables explain the patterns of political consolidation and rebellion that we observe in medieval Europe?

First, note that greater resources have a straightforward effect on political consolidation. Innovations that improve agricultural productivity, such as the adoption of the iron plough in Northern Europe c. 800–1200 AD (Anderson, Jensen, and Skovsgaard, 2016), increase e_i and, by our model, would have led to great levels of consolidation among elites. The medieval warming period after 1100 also increased productivity and raised e_i, thereby facilitating consolidation. Conversely, periods of economic crisis such as the Great Famine of 1317–1320 would have prompted rebellions by elites. And indeed the following decade saw a series of rebellions and conflicts in England, beginning with the rebellion of Thomas of Lancaster against the favorites of Edward II (r. 1307–1327), and culminating with the successful rebellion of Roger Mortimer and Queen Isabella in 1326.

These predictions relating to the size of resources are common to many explanations of political order and disorder. The more distinctive predictions that come from our model pertain to the non-appropriability of resources and the fighting strength of the weakest members. They suggest that the logic we have for studying centralized states does not apply to the feudal world.

Take, for instance, the claim that consolidation in modern and early modern period was associated with greater state capacity and the power of the ruler relative to elites (see Gennaioli and Voth, 2015; Johnson and Koyama, 2017). Historians associate the rise of the modern state with the military revolution and the growth of professional standing armies (Parker, 1976). This reasoning may not apply, however, in the feudal period. Our model suggests that under feudalism, consolidation occurs not because of the rise of centralized power, but because the king is willing to take a smaller share of resources so as to prevent the rebellion of even the weakest, least militarily valuable, elites.

Another example is the relationship between the observability of resources and political consolidation. Mayshar et al. (2017), Scott (2017), and Mayshar et al. (2022) argue that the presence of agricultural crops whose output is highly observable and hence appropriable by the ruler, such as wheat, leads to the formation of stronger states. This logic applies to the rise of early states reliant on taxation. But this reasoning does not hold in the feudal world precisely because the ruler relied on contributions of elites, rather than tax revenues. The ruler, therefore, had to continuously bargain with elites. In this environment, a lord or baron whose resources are easily appropriable by the king poses less threat of rebellion. If he rebelled, he would only be able to take back a small amount of resources from the king. The king, therefore, would be more willing to violate existing agreements with the lord, e.g. give the lord less than his share of the coalition's resources to keep for himself and allocate to other members. The lord would rebel and exit the coalition, but since he cannot take much, the total resources of the coalition would not decrease significantly and, in addition, there would be one less member (the rebel) with which these resources would have to be shared. The king, therefore, prefers to let that lord rebel. (In contrast, a lord whose resources are non-appropriable is more costly to lose, and therefore prompts the king to keep sharing the coalition's total resources with that lord to keep him loyal.) Thus, in a feudal world, the appropriability of resources generates more rebellion and therefore makes consolidation *less*, not more, likely.[22]

Theorem 2 and its corollary also predict that consolidation into a single feudal coalition is more likely when the military capabilities of the weakest elite members are sufficiently high. This has important implications for how we understand episodes of political consolidation under feudalism.

Feudalism is associated with military technologies that favored landed elites such as the stirrup and with economic developments that ensured that central states remained weak (and unable to raise substantial taxes) (e.g. Beeler, 1971, 9–10),[23] That is, the available military and economic technology ensured that the distribution of military capabilities was highly egalitarian. Particularly in the 11th and 12th centuries, the state of military technology favored defense. A single well-fortified lord could resist a king with a much larger army for many months (therefore forcing him to spend tremendous resources in a long siege).[24] Under these circumstances, when even the weakest member of the elite had sufficient military power, the king will distribute resources in such a way so as to keep all members of the elite within his coalition.

Lastly, our model can generate predictions with respect to the particular role of castles in a feudal environment. Historians often associate the presence of castles with the fracturing of political authority in the medieval world (e.g Power, 1999, 110–111).[25] In a similar vein, recent work by Cappelen and Hariri (2022) argues that private or baronial castles are a measure of state weakness. The ratio of royal to private castles measures the extent to which a state has obtained a Weberian monopoly of violence within a kingdom. State building then required centralized monarchs to demolish private castles. We agree that this reasoning does apply to the early modern period, when the fortifications of the nobility *were* a barrier to political consolidation and centralization and as result dismantled or destroyed.

However, this logic is alien to a feudal environment. Feudal monarchs did not seek to dismantle the castles of their lords, even when these fortifications increased the ability of the lords to withdraw resources from the king. In the feudal world, the strength of the lords reflected the glory of the king. The role of castles is therefore the opposite of Weberian logic: the presence of castles should be associated with more

[22] Recognizing that feudal polities followed a fundamentally different logic to modern states helps make sense of common misunderstandings. For instance, modern historians question the usefulness of the term “The Angevin Empire” in reference to the assorted parts of France and England ruled by Henry II (r. 1154–1189) and his sons (Gillingham, 2001, 4) They note that it had no common or centralized administrative structure nor was it a unified territorial state. Furthermore, it collapsed rapidly once King John (r. 1199–1216) was defeated by the French King Philip Augustus (r. 1180–1223). The transient nature of this polity makes sense once one recognizes that it comprised a coalition of military elites.

[23] This thesis is most strongly associated with White (1962). For a modern perspective, see Bachrach and Bachrach (2017).

[24] Levine and Modica (2022) also associate strong defensive fortifications with political institutions that favor the military elite.

[25] Describing the same period, Holland (2008, 141) writes: “The Capetians, as they struggled to assert their authority over even the patchwork of territories that constituted the royal domain, were hardly in any position to forbid distant princes from raising fortifications of their own. The consequences, sprouting up suddenly across region after region of West Francia, like toadstools from rotten wood, was a great host of strange and unsettling structures, as menacing as they were crude: what would come to. be termed in English ‘castles.”’

and not less (feudal) consolidation. In Norman England after the Conquest, castles were built across the country: by 1154 there were 225 baronial castles (compared to 49 royal castles) in England (Brown, 1959, 249).[26] Castles, by increasing the security of elites, could consolidate a feudal realm by making them more credible partners of the king.

Whenever there were rebellions, they generally occurred when the ruler extracted too much from the elites and therefore violated existing sharing arrangements. King John faced rebellion largely because "[t]he barons...resented paying more" (Turner and John, 2005, 69). Richard II (r. 1377–1399) similarly alienated the political elite and was deposed for this reason. One can also interpret the failure of a king to do his duties as a form of non-monetary extraction. This would account for the failure of Henry VI (r. 1422–1461) which led to the Wars of the Roses.

More generally, our model suggests that a greater probability of rebellion could result from changes in military technology that affected the extent to which resources can be appropriated by the ruler. Technological innovations like the trebuchet introduced at the end of the 12th century shifted the balance of power towards besiegers (Gravett, 1990, 49–51). But trebuchets required trained engineers and were expensive to construct, so this innovation disproportionately benefited kings. Our model predicts that, counter-intuitively, such innovations would have increased the number of rebellions. The logic is simple: technologies that strengthened the power of the king would embolden him to violate existing sharing arrangements with elites, generating more rebellions in equilibrium. King John, again, was the first English king to deploy trebuchets on a large scale.

6 Concluding comments

In this paper we depict a political economy in which there is no state with a monopoly of violence. Rather, there are elites endowed with resources, both economic and military, who can form alliances in a peaceful way or through battles and conquest. Alliances are also non-binding in that parties can rebel. This political economy captures key features of the feudal world and is relevant for thinking about political order in situations of anarchy or fragmented and weak states.

In such an environment, we derive conditions under which alliances are stable and eventually lead to a consolidated realm, in which all the elites belong to, and stay in, one grand coalition. Consolidation is more likely when elites' resources are large and mostly non-appropriable by the ruler, and when the weakest fighters have sufficiently high military capability.

Our model of the feudal world poses a direct contrast to existing notions of early states. Most notably, while existing literature have shown that states are more likely consolidated when resources can be more easily appropriated by the ruler, we demonstrate the opposite. When a ruler has no independent resources to tax, she relies instead on the economic and military contributions of feudal lords, and therefore has to form

[26] Strong kings like Henry II did destroy or occupy many baronial castles. But they also permitted their favored barons to build new castles: "Alienations of royal castles …are not infrequent to those whose support at any given moment was trusted or needed" (Brown, 1959, 256)

a coalition with them in order to rule. This requires the ruler to share the coalition's resources among its members. Consolidation is thus achieved when sharing agreements are always kept; otherwise, rebellions occur and the realm is fragmented. Resources that are *not* appropriable by the ruler enable rebels to seize them from the coalition, making rebellion more costly to the ruler. This, then, induces the ruler to honor existing agreements in order to prevent rebellion, and the realm is consolidated.

Theory appendix

In Appendix Section 7, we present additional results that are used to construct the proofs. Appendix Section 8 contains the actual proofs.

7 Additional results

Optimal action pairs

To construct an FPE equilibrium, one needs to construct optimal action pairs. For this purpose, we elaborate on the expected payoffs.

From Definition 1, we know that if a proposer-responder action pair is optimal, then $\lambda_k = 1$ when $V^k(\mu_j, \lambda_k = 1, s) > V^k(\mu_j, \lambda_k = 0, s)$. From (1) and (2), the latter condition is more likely to hold when the differences between $V^k(\mu_j = 1, \lambda_k = 1, s_{LA})$ and $V^k(\mu_j = 1, \lambda_k = 0, s_{LF})$, and between $V^k(\mu_j = 0, \lambda_k = 1, s_{RA})$ and $V^k(\mu_j = 0, \lambda_k = 0, s_{RF})$, are large.

We first look at the difference between $V^k(\mu_j = 0, \lambda_k = 1, s_{RA})$ and $V^k(\mu_j = 0, \lambda_k = 0, s_{RF})$. Without loss of generality, let $j = 1$ and let responders be drawn to play sequentially, i.e. $k = 2$ at $t = 1$, $k = 3$ at $t = 2$, etc.

For responder k drawn to play at $t = 1$, one can construct $V^k(\mu_j = 1, \lambda_k = 1, s_{LA}) = u^k(s_{LA})_0 + \delta u^k(s_{LA})_1 + \delta^2 u^k(s_{LA})_2 + \delta^3 u^k(s_{LA})_3 + \dots$ or, letting $j = 1$ and $k = 2$:

$$V^2(\mu_1 = 1, \lambda_2 = 1, s_{LA}) = e_2 + \delta\alpha_{2,1}(e_1 + e_2) + \delta^2\alpha_{2,2}(e_1 + e_2 + e_3) + \delta^3\alpha_{2,3}(e_1 + e_2 + e_3 + e_4) + \dots, \quad (4)$$

where e_2 is $k = 2$'s resources which she owns entirely prior to joining $j = 1$'s coalition, $\alpha_{2,1}$ is 2's share of the coalition's resources at $t = 1$, which is the sum of 1 and 2's resources, and $\alpha_{2,2}, \alpha_{2,3}, \dots$ are analogously defined. The coalition's resources grow with each draw of responder since under state s_{LA}, each responder drawn joins the coalition (peacefully).

Now, under state s_{LF}, each responder drawn to play fights with j, but is conquered because j induces loyalty among the approval committee. Thus, each responder enters the coalition, but bears cost of fighting c_k. That is, its resources shrink by amount c_k at the period of joining. Cost c_k is temporary, and k's resources are replenished and grows back to e_k by the start of the next period.

A member of the approval committee also incurs cost of fighting $c_i \mathbb{1}_{A_t}$, where $\mathbb{1}_{A_t}$ is an indicator variable equal to one if the player is in the approval committee at t. The coalition fighting cost needed to conquer k at t is thus $C_t = \sum c_i \mathbb{1}_{A_t}$.[27]

Thus, for responder k drawn to play at $t = 1$, one can construct $V^k(\mu_j = 1, \lambda_k = 0, s_{LF}) = u^k(s_{LF})_0 + \delta u^k(s_{LF})_1 + \delta^2 u^k(s_{LF})_2 + \delta^3 u^k(s_{LF})_3 + \ldots$ or, letting $j = 1$ and $k = 2$:

$$\begin{aligned} V^2(\mu_1 = 1, \lambda_2 = 0, s_{LF}) &= e_2 + \delta\alpha_{2,1}\Big((e_1 - c_1) + (e_2 - c_2)\Big) \\ &\quad + \delta^2\alpha_{2,2}\Big((e_1 - c_1\mathbb{1}_{A_2}) + (e_2 - c_2\mathbb{1}_{A_2}) + (e_3 - c_3)\Big) \\ &\quad + \delta^3\alpha_{2,3}\Big((e_1 - c_2\mathbb{1}_{A_3}) + (e_2 - c_2\mathbb{1}_{A_3}) + (e_3 - c_3\mathbb{1}_{A_3}) \\ &\quad + (e_4 - c_4)\Big) + \ldots, \end{aligned} \tag{5}$$

where note that at the start of the first period, $j = 1$ is necessarily in the approval committee as she is the only member of the coalition at $t = 0$. Thus, she always incurs fighting cost in the first period if she goes to war with k. Also note that the responder always incurs fighting costs at the time of joining the coalition — at $t = 1$, $k = 2$ bears cost c_2, at $t = 2$, $k = 3$ bears cost c_3, etc.

With non-zero coalition fighting costs, (4) is always greater than (5). Thus, all else equal, the greater the (positive) difference between (4) and (5), the more likely it is that $k = 2$ chooses $\lambda_2 = 1$.

We next look at the difference between $V^k(\mu_j = 0, \lambda_k = 1, s_{RA})$ and $V^k(\mu_j = 0, \lambda_k = 0, s_{RF})$. One can construct $V^k(\mu_j = 0, \lambda_k = 1, s_{RA}) = u^k(s_{RA})_0 + \delta u^k(s_{RA})_1 + \delta^2 u^k(s_{RA})_2 + \delta^3 u^k(s_{RA})_3 + \ldots$ or, letting $j = 1$ and $k = 2$:

$$V^2(\mu_1 = 0, \lambda_2 = 1, s_{RA}) \tag{6}$$

$$\begin{aligned} &= e_2 + \delta\alpha_{2,1}\Big(e_1 + e_2 - (e_1 - r_1)\Big) \\ &\quad + \delta^2\alpha_{2,2}\Big(e_1 + e_2 + e_3 - (e_1 - r_1)\mathbb{1}_{R_2} - (e_2 - r_2)\mathbb{1}_{R_2}\Big) \\ &\quad + \delta^3\alpha_{2,3}\Big(e_1 + e_2 + e_3 + e_4 - (e_1 - r_1)\mathbb{1}_{R_3} - (e_2 - r_2)\mathbb{1}_{R_3} - (e_3 - r_3)\mathbb{1}_{R_3}\Big) \\ &\quad + \ldots, \end{aligned} \tag{7}$$

where r_i is i's cost of rebellion, $\mathbb{1}_{R_t}$ an indicator variable equal to one if i rebels from the coalition at t, with $R_t \subseteq A_t$ denoting the set of approval committee members who rebel. Like the cost of fighting, r_i is temporary and is thus only incurred at the time of rebellion. Thus, the resources of a rebel shrink at the time of rebellion but is fully replenished at the start of the next period. A rebel then takes from the coalition $e_i - r_i$ at the time of rebellion. One can then interpret r_i as the appropriable portion of e_i that i cannot take back from the coalition. Note that under state s_{RA}, only $j = 1$ rebels in

[27] That c_i is fixed per period is without loss of generality – what matters is total cost of fighting C_t against k which varies by period. Thus, how C_t is shared by the coalition members is also immaterial. C_t is large, for instance, when the approval committee has many members with large individual fighting costs.)

the first period, taking away $e_1 - r_1$ from the coalition. (Similarly, rebellion cost r_j captures the appropriable portion of e_j that j cannot take away from the coalition.) At any time period thereafter, any member of the approval committee can rebel, which excludes the new responder who peacefully accepts the proposal.

Lastly, one can construct $V^k(\mu_j = 0, \lambda_k = 0, s_{RF}) = u^k(s_{RF})_0 + \delta u^k(s_{RF})_1 + \delta^2 u^k(s_{RF})_2 + \delta^3 u^k(s_{RF})_3 + \ldots$ or, letting $j = 1$ and $k = 2$:

$$V^2(\mu_1 = 0, \lambda_2 = 0, s_{RF}) = e_2 + \delta(e_2 - c_2) + \delta^2 e_2 + \delta^3 e_2 + \ldots, \tag{8}$$

where $k = 2$ incurs temporary fighting cost at $t = 1$, i.e. when she is drawn to play and fights with $j = 1$. Thereafter, she keeps her entire resources e_2 since she is outside the coalition.

All else equal, the greater the (positive) difference between (6) and (8), the more likely it is that $k = 2$ chooses $\lambda_2 = 1$. Note that (6) is not always larger than (8), but a positive and large difference becomes more likely, when $k = 2$'s resources are small, coalition members' resources are large, and the costs of rebellion are large.

Finally, note that equations (4) to (8) are generalizable to any responder k, and for any order of responders. (One simply changes notation – superscript 2 in $V^2(\cdot)$ and subscript 2 in $\{\alpha_{2,t}\}$ to any k, and the subscripts for the other variables can be easily changed to reflect the order of responders. Similarly for any j, and any order in which j is drawn.)

Next, recall that an optimal action pair also requires $\mu_j = \arg\max V^j(\mu_j, \lambda_k, s)$. We then elaborate on $V^j(\mu_j, \lambda_k, s)$. First, we construct, for $j = 1$ playing at $t = 1$, and assuming a sequential draw of responders, i.e. $k = 2$ at $t = 1$, $k = 3$ at $t = 2$, etc, the following:

$$\begin{aligned} V^1(\mu_1 = 1, \lambda_2 = 1, s_{LA}) &= e_1 + \delta\alpha_{1,1}(e_1 + e_2) + \delta^2\alpha_{1,2}(e_1 + e_2 + e_3) \\ &\quad + \delta^3\alpha_{1,3}(e_1 + e_2 + e_3 + e_4) + \ldots \end{aligned} \tag{9}$$

$$\begin{aligned} V^1(\mu_1 = 0, \lambda_2 = 1, s_{RA}) &= e_1 + \delta\alpha_{1,1}\Big(e_1 + e_2 - (e_1 - r_1)\Big) \\ &+ \delta^2\alpha_{1,2}\Big(e_1 + e_2 + e_3 - (e_1 - r_1)\mathbb{1}_{R_2} - (e_1 - r_1)\mathbb{1}_{R_2}\Big) \\ &+ \delta^3\alpha_{1,3}\Big(e_1 + e_2 + e_3 + e_4 - (e_1 - r_1)\mathbb{1}_{R_3} - (e_2 - r_2)\mathbb{1}_{R_3} - (e_3 - r_3)\mathbb{1}_{R_3}\Big) \\ &+ \ldots, \end{aligned} \tag{10}$$

$$\begin{aligned} V^1(\mu_1 = 1, \lambda_2 = 0, s_{LF}) &= e_1 + \delta\alpha_{1,1}\Big((e_1 - c_1) + (e_2 - c_2)\Big) \\ &+ \delta^2\alpha_{1,2}\Big((e_1 - c_1\mathbb{1}_{A_2}) + (e_2 - c_2\mathbb{1}_{A_2}) + (e_3 - c_3)\Big) \\ &+ \delta^3\alpha_{1,3}\Big((e_1 - c_1\mathbb{1}_{A_3}) + (e_2 - c_2\mathbb{1}_{A_3}) + (e_3 - c_3\mathbb{1}_{A_3}) + (e_4 - c_4)\Big) + \ldots, \end{aligned} \tag{11}$$

$$V^1(\mu_1 = 0, \lambda_2 = 1, s_{RF}) = e_1 + \delta(e_1 - r_1 - c_1) + \delta^2(e_1 - r_1 - c_1) + \delta^3(e_1 - r_1 - c_1) + \ldots \tag{12}$$

Now suppose $k = 2$ were to accept $j = 1$'s proposal. If $\lambda_2 = 1$, (3) implies that $j = 1$ would choose $\mu_1 = 1$ if $V^1(\mu_1 = 1, \lambda_2 = 1, s_{LA}) > V^1(\mu_1 = 0, \lambda_2 = 1, s_{RA})$; $\mu_1 = 0$ if the reverse inequality holds, and $\mu_1 \in [0, 1]$ if equality holds. Because of non-zero costs of rebellion, (9) is always greater than (10).

Now suppose that $k = 2$ were to reject $j = 1$'s proposal. If $\lambda_2 = 0$, (3) implies that $j = 1$ would choose $\mu_1 = 1$ if $V^1(\mu_1 = 1, \lambda_2 = 0, s_{LF} > V^1(\mu_1 = 0, \lambda_2 = 0, s_{RF}$; $\mu_1 = 0$ if the reverse inequality holds, and $\mu_1 \in [0, 1]$ if equality holds. We thus compare (11) and (12).

Note that (11) may be less than (12), but a positive and large difference becomes more likely, if $j = 1$'s resources are small; its costs of fighting and of rebellion are large; coalition members' resources are large and their costs of fighting small.

Finally, note that equations (9) to (12) easily generalize to any j and any order of responder, simply by changing the relevant superscript and subscripts.

We can now characterize an optimal action pair, using the following sets of thresholds, conditions, and cut-off points.

Definition 5 The **responder threshold** is a collection of (minimum) values $\{\underline{e_j}_k, \{\underline{e_k}_k\}, \underline{r_j}_k, \{\underline{r_k}_k\}, \underline{c_k}_k\} \equiv \{e_j, \{e_k\}, r_j, \{r_k\}, c_k\}$ such that, given state s and $\{\alpha_{k,t}\}$, $V^k(\mu_j = 0, \lambda_k = 1, s_{RA}) = V^k(\mu_j = 0, \lambda_k = 0, s_{RF})$.

That is, at the responder threshold, the responder is indifferent between accepting or rejecting the proposal, given that there will be rebellion.

Definition 6 The **proposer threshold** is a collection of (maximum) values $\{\{\bar{C_t}\}_j, \bar{c_k}_j\} \equiv \{\{C_t\}, c_k\}$ and (minimum) value $\underline{r_j}_j \equiv r_j$ such that, given s and $\{\alpha_{j,t}\}$, $V^j(\mu_j = 1, \lambda_k = 0, s_{LF}) = V^j(\mu_j = 0, \lambda_k = 1, s_{RF})$.

That is, at the proposer threshold, the proposer is indifferent between inducing loyalty or rebellion, given that the responder will fight.

Key to obtaining an optimal action pair is whether or not these thresholds are met. Consider conditions (a) and (b) below:

Definition 7 **Condition (a)** is met if every element in $\{e_j, \{e_k\}, r_j, \{r_k\}, c_k\}$ is greater than or equal to its respective threshold value in $\{\underline{e_j}_k, \{\underline{e_k}_k\}, \underline{r_j}_k, \{\underline{r_k}_k\}, \underline{c_k}_k\}$.

Definition 8 **Condition (b)** is met if every element in $\{\{C_t\}, c_k\}$ is less than or equal to its respective threshold value in $\{\{\bar{C_t}\}_j, \bar{c_k}_j\}$, and r_j is greater than or equal to threshold value $\underline{r_j}_j$.

If conditions (a) and (b) are not met, then the following cut-off points become relevant.

Definition 9 Suppose that $V^k(\cdot, s_{RA}) < V^k(\cdot, s_{RF})$. Then the **responder's cut-off** is $\underline{\mu_j} \in \{\mathbb{R} > 0\} \equiv \mu_j$ such that

$$\begin{aligned}&\mu_j V^k(\mu_j = 1, \lambda_k = 1, s_{LA}) + (1 - \mu_j) V^k(\mu_j = 0, \lambda_k = 1, s_{RA})\\&\quad = \mu_j V^k(\mu_j = 1, \lambda_k = 0, s_{LF})\\&\qquad + (1 - \mu_j) V^k(\mu_j = 0, \lambda_k = 0, s_{RF}).\end{aligned}$$

Definition 10 Suppose that $V^j(\cdot, s_{LF}) < V^j(\cdot, s_{RF})$. Then the **proposer's cut-off** is $\underline{\lambda_k} \in \{\mathbb{R} > 0\} \equiv \lambda_k$ such that

$$\begin{aligned}&\lambda_k V^j(\mu_j = 1, \lambda_k = 1, s_{LA}) + (1 - \lambda_k) V^j(\mu_j = 1, \lambda_k = 0, s_{LF})\\&\quad = \lambda_k V^j(\mu_j = 0, \lambda_k = 1, s_{RA})\\&\qquad + (1 - \lambda_k) V^j(\mu_j = 0, \lambda_k = 0, s_{RF}).\end{aligned}$$

One can then construct an optimal action pair using conditions (a) and (b), and the responder's and proposer's cut-offs:

Lemma 2 *A pair (μ_j, λ_k) of proposer-responder actions for $i = \{j, k\}$ is an optimal action pair if:*

1. $\lambda_k = 1$ *if condition (a) holds. If (a) does not hold:*

$$\lambda_k = \begin{cases} 1 & \text{if } \mu_j > \underline{\mu_j} \\ [0, 1] & \text{if } \mu_j = \underline{\mu_j} \\ 0 & \text{if } \mu_j < \underline{\mu_j} \end{cases}$$

2. $\mu_j = 1$ *if condition (b) holds. If (b) does not hold:*

$$\mu_j = \begin{cases} 1 & \text{if } \lambda_k > \underline{\lambda_k} \\ [0, 1] & \text{if } \lambda_k = \underline{\lambda_k} \\ 0 & \text{if } \lambda_k < \underline{\lambda_k}, \end{cases}$$

Proof (All proofs are in section 2 of the Theory Appendix).

Pairwise alliance

The following result groups optimal action pairs into four types.

Lemma 3 *There exist only four types of optimal action pairs, each obtained by four sets of conditions. For any $i = \{j, k\}$, the optimal action pair (μ_j, λ_k) is determined by the following:*

1. *If* ***condition (a) holds and*** $\underline{\lambda_k} < 1$***, or condition (b) holds and*** $\underline{\mu_j} < 1$***, or both (a) and (b) hold****, then* $(\mu_j = 1, \lambda_k = 1)$.

2. *If* ***condition (a) does not hold, condition (b) holds, and*** $\underline{\mu}_j \geq 1$, *then* $(\mu_j = 1, \lambda_k \in [0, 1))$.
3. *If* ***condition (a) holds, condition (b) does not hold, and*** $\underline{\lambda}_k \geq 1$, *then* $(\mu_j \in [0, 1), \lambda_k = 1)$.
4. *If* ***condition (a) does not hold and condition (b) does not hold***, *then* $(\mu_j = 0, \lambda_k = 0)$.

In turn, the conditions in Lemma 3 are affected by some key variables, to wit:

Corollary 2 *The following variables determine the likelihood that condition (a) and condition (b) are met, and whether* $\underline{\lambda}_k \geq 1$ *and* $\underline{\mu}_j \geq 1$: $e_j, \{e_k\}, r_j, \{r_k\}, c_k, \{C_t\}$. *Specifically:*

1. ***Resources of*** j, e_j, ***and of each*** k, $\{e_k\}$:
The larger e_j *is, the more likely that (a) is met, and the less likely that* $\underline{\mu}_j \geq 1$. *The larger* $\{e_k\}$ *are, the more likely that (a) is met, the less likely that* $\underline{\lambda}_k \geq 1$, *and the less likely that* $\underline{\mu}_j \geq 1$.
2. ***Appropriability of*** j***'s resources,*** r_j, ***and of each of*** k***'s,*** $\{r_k\}$:
The larger r_j *is, the more likely that (a) is met, the more likely that (b) is met, the more likely that* $\underline{\lambda}_k \geq 1$, *and the less likely that* $\underline{\mu}_j \geq 1$. *The larger* $\{r_k\}$ *are, the more likely that (a) is met, the more likely that* $\underline{\lambda}_k \geq 1$, *and the less likely that* $\underline{\mu}_k \geq 1$.
3. ***Cost of fighting of*** k, c_k, ***and of the coalition at each*** t, $\{C_t\}$:
The larger c_k *is, the more likely that (a) is met, the less likely that (b) is met, the more likely that* $\underline{\lambda}_k \geq 1$, *and the less likely that* $\underline{\mu}_j \geq 1$. *The larger* $\{C_t\}$ *are, the less likely that (b) is met, the more likely that* $\underline{\lambda}_k \geq 1$, *and the less likely that* $\underline{\mu}_j \geq 1$.

The following table summarizes the effect of each variable on the likelihood that each key restriction is met:

	condition (a)	*condition (b)*	$\underline{\lambda}_k \geq 1$	$\underline{\mu}_j \geq 1$
e_j	$\uparrow$			$\downarrow$
$\{e_k\}$	$\uparrow$		$\downarrow$	$\downarrow$
r_j	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\{r_k\}$	$\uparrow$		$\uparrow$	$\downarrow$
c_k	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\{C_t\}$		$\downarrow$	$\uparrow$	$\downarrow$

We now use the above results to assess the likelihood that a pair of players, when drawn to play, successfully form an alliance. To do this, recall that the FPE equilibrium entails that all players choose optimal action pairs. Thus, in equilibrium, the pairwise outcome – the actions chosen by a randomly drawn pair of players, is also determined by the same sets of conditions that determine the optimal action pair. There are thus four types of pairwise outcomes, corresponding to each set of conditions. That is, denoting a pairwise outcome as $[\mu_j, \lambda_k]$ (to distinguish it from optimal action pair (μ_j, λ_k)), there are also four types:

Proposition 3 ***Pairwise Outcomes and Conditions*** *In equilibrium, the outcome from any pairwise play can be any of the following:*

1. ***Peaceful Alliance*** *between j and k, i.e. $[\mu_j = 1, \lambda_k = 1]$ requires* ***only condition (a) to hold and $\underline{\lambda_k} < 1$, or only condition (b) to hold and $\underline{\mu_j} < 1$, or both (a) and (b) to hold.***
2. ***Alliance by Conquest*** *of k by j, i.e. $[\mu_j = 1, \lambda_k \in [0, 1)]$ requires* ***condition (a) not to hold, condition (b) to hold, and $\underline{\mu_j} \geq 1$.***
3. ***Alliance with Unrest*** *in which k accepts to join j's coalition, but some members rebel i.e. $[\mu_j \in [0, 1), \lambda_k = 1]$, requires* ***condition (a) to hold, condition (b) not to hold, and $\underline{\lambda_k} \geq 1$.***
4. ***No Alliance*** *between j and k, i.e. $[\mu_j = 0, \lambda_k = 0]$, requires* ***condition (a) and condition (b) not to hold.***

It follows from Lemma 3 and Corollary 2 that the the size of resources, the extent of their appropriability, and the costs of fighting also affect the likelihood that any of the four pairwise outcomes is obtained from any pairwise play. (See Theorem 1).

Consolidation

The following result establishes the necessary and sufficient condition that generates a consolidation equilibrium.

Proposition 4 ***Consolidation Equilibria*** *A consolidation equilibrium is obtained if, for all $i = \{j, k\}$, either of the following is true:*

1. *condition (b) holds, or*
2. *if (b) does not hold, $\underline{\lambda_k} < 1$.*

In turn, whether these conditions are met depends on the same key variables that determine the type of optimal action pairs and pairwise outcomes. These variables, therefore, affect the likelihood of consolidation. The precise manner is established in Theorem 2.

8 Proofs

Proof of Lemma 1

Assumption 1 readily implies that in an equilibrium in which i is in the coalition at t, $\alpha_{i,t} = \underline{\alpha_i}$. This is because, given that i is in the coalition and therefore is allocated at least share $\underline{\alpha_i}$, any share above this reservation share takes away from j's share. Since the expected payoffs to pure actions (equations 4 to 12) entail allotting reservation shares to the non-ruling members of the coalition, this also holds for all mixed actions and, hence, at each t.

Note, then, that in equation (4), the shares that k obtains at each t are $\alpha_{2,1} = \alpha_{2,2} = \alpha_{2,3} = ... = \underline{\alpha_2}$. Similarly, in (5) and in (6), $\alpha_{2,1} = \alpha_{2,2} = \alpha_{2,3} = ... = \underline{\alpha_2}$.

The shares for j, however, change whenever a member joins or exits the coalition, since j gets all the remaining share, i.e. 1 minus the sum of the reservation shares of the coalition members at t. This implies that $\alpha_{1,1}$ is the same across equation (8), (9), (10), $\alpha_{1,2}$ is the same across (8), (9), (10), etc., but where $\alpha_{1,1} > \alpha_{1,2} > \alpha_{1,3} > \ldots$, since j relinquishes some share as new members join the coalition. Note, however, that because $\sum \alpha_i \le 1$, with $i \in N$ and N including the ruler j, then $(1 - \sum \alpha_{i_\omega}) \ge \underline{\alpha}_j$.

We thus impose the above shares in constructing all the other proofs.

Proof of Lemma 2

Suppose condition (a) holds. Then (6) is greater than or equal to (8). Because (4) is always greater than (5), then $\lambda_k = 1$ for any $\mu_j \in [0, 1]$ if (a) holds. If (a) does not hold, then $\lambda_k = 1$ is the best response to any $\mu_j \in (\overline{\mu_j}, 1]$. Otherwise, for $\mu_j \in [0, \overline{\mu_j})$, $\lambda_k = 0$ is the best response of k. Lastly, if $\mu_j = \overline{\mu_j}$, then $\lambda_k \in [0, 1]$.

Now suppose condition (b) holds. Then (11) is greater than or equal to (12). Because (9) is always greater than (10), $\mu_j = 1$ for any $\lambda_k \in [0, 1]$ if (b) holds. If (b) does not hold, then $\mu_j = 1$ is the best response to any $\lambda_k > \underline{\lambda_k}$. It also follows that $\mu_j \in [0, 1]$ is the best response to $\lambda_k = \underline{\lambda_k}$, and $\mu_j = 0$ to any $\lambda_k < \underline{\lambda_k}$.

Proof of Lemma 3

First note that the conditions in (1) to (4) are exhaustive, since condition (a) can hold or not hold, condition (b) can hold or not hold, $\underline{\lambda_k}$ can be less than one or greater than or equal to 1, and $\overline{\mu_j}$ can be less than one or greater than or equal to 1. These generate the corresponding outcomes in (1) to (4), which are also exhaustive in that they include all the possible combinations of all possible actions taken by j and k. Specifically, j can choose to offer a proposal that will not induce any rebellion, $\mu_j = 1$, or that will certainly induce rebellion, $\mu_j = 0$, or that will induce rebellion with some non-zero probability, $\mu_j \in [0, 1)$. Similarly, k can choose not to fight, $\lambda_k = 1$, to certainly fight, $\lambda_k = 0$, or to fight with some non-zero probability, $\lambda_k \in [0, 1)$.

The proof makes use of Lemma 2.

We first prove (1). When condition (a) holds, then from Lemma 2, $\lambda_k = 1$ is the dominant action for k. When condition (b) holds, then $\mu_j = 1$ is the dominant action for j. Thus, when (a) and (b) both hold, the outcome is $(\mu_j = 1, \lambda_k = 1)$.

When (b) does not hold, $\mu_j = 1$ is j's best response to any $\lambda_k > \underline{\lambda_k}$. The latter implies, given k chooses $\lambda_k = 1$, which is its dominant action if (a) holds, that $\underline{\lambda_k} < 1$. Thus, when (a) holds, (b) does not, and $\underline{\lambda_k} < 1$, the outcome is also $(\mu_j = 1, \lambda_k = 1)$.

When (a) does not hold, $\lambda_k = 1$ is k's best response to any $\mu_j > \overline{\mu_j}$, which implies that $\overline{\mu_j} < 1$, given that j chooses $\mu_j = 1$, which is its dominant action if (b) holds. Thus, when (a) does not hold, (b) holds, and $\overline{\mu_j} < 1$, the outcome is also $(\mu_j = 1, \lambda_k = 1)$.

We next prove (2). The outcome $(\mu_j = 1, \lambda_k \in [0, 1))$ cannot be obtained if condition (a) holds since this would make $\lambda_k = 1$ the dominant action for k. It can only be obtained when (a) does not hold and $\mu_j \le \overline{\mu_j}$, since this induces $\lambda_k \in [0, 1]$ or $\lambda_k = 0$. Given $\mu_j = 1$ (since (b) holds), it must then be that $\overline{\mu_j} \ge 1$. Thus, when condition (a) does not hold, condition (b) holds, and $\underline{\mu_j} \geq 1$, the outcome is $(\mu_j = 1, \lambda_k \in [0, 1))$.

We next prove (3). The outcome $(\mu_j = 1, \lambda_k = 1)$ cannot be obtained if condition (b) holds since this would make $\mu_j = 1$ the dominant action for j. It can only be obtained when (b) does not hold and $\lambda_k \leq \underline{\lambda_k}$, since this induces $\mu_j \in [0, 1]$, or $\mu_j = 0$. Given $\lambda_k = 1$ (since (a) holds), it must then be that $\underline{\lambda_k} \geq 1$. Thus, when condition (a) holds, condition (b) does not hold, and $\underline{\lambda_k} \geq 1$, the outcome is $(\mu_j = 1, \lambda_k = 1)$.

Lastly, we prove (4). The outcome $(\mu_j = 0, \lambda_k = 0)$ cannot be obtained if condition (a) or (b) holds, for this would make $\lambda_k = 1$ or $\mu_j = 1$ the respective dominant action for k and j. To show that there is no need to place restrictions on $\underline{\lambda_k}$ or $\underline{\mu_j}$, note that with (a) not holding, $\lambda_k = 0$ requires $\mu_j < \underline{\mu_j}$. The latter, however, is already satisfied, since $\lambda_k = 0$ already prompts j to choose $\mu_j = 0$ for any $\underline{\lambda_k} > 0$. Similarly, with (b) not holding, $\mu_j = 0$ requires $\lambda_k < \underline{\lambda_k}$, which is already satisfied since $\mu_j = 0$ already prompts k to choose $\lambda_k = 0$ for any $\underline{\mu_j} > 0$.

Proof of Corollary 2

The larger e_1 is, the more likely that it surpasses threshold $\underline{e_1}_k$ and thus, from Definition 5 (D.5), that condition (a) is met. It also decreases the difference between equation (8) and (6) and thus, from D.7 and Lemma 2, makes it less likely that $\underline{\mu_j} \geq 1$ (given $V^k(\cdot, s_{RA}) < (V^k(\cdot, s_{RF}))$.

The larger $\{e_k\}$ are, the more likely that they surpass their respective thresholds in $\{\underline{e_k}_k\}$ and thus, from D.5, that (a) is met. It also decreases the difference between equations (12) and (11) and thus, from D.7 and Lemma 2, makes it less likely that $\underline{\lambda_k} \geq 1$ (given $V^j(\cdot, s_{LF}) < V^j(\cdot, s_{RF})$). Lastly, it decreases the difference between (8) and (6) and thus, from D.7 and Lemma 2, makes it less likely that $\underline{\mu_j} \geq 1$ (given $V^k(\cdot, s_{RA}) < V^k(\cdot, s_{RF})$.)

The larger r_1 is, the more likely that it surpasses threshold $\underline{r_1}_j$ and thus, from D.6, that condition (b) is met. It also increases the difference between (12) and (11) and thus, from D.8 and Lemma 2, makes it more likely that $\underline{\lambda_k} \geq 1$ is met (given $V^j(\cdot, s_{LF}) < V^j(\cdot, s_{RF})$). Lastly, it decreases the difference between (8) and (6) and, thus, from D.5 and Lemma 2, makes it less likely that $\underline{\mu_j} \geq 1$.

The larger $\{r_k\}$ are, the more likely that they surpass their respective thresholds in $\{\underline{r_k}_k\}$ and thus, from D.5, that (a) is met. It also decreases the difference between (9) and (10) and thus, from D.7 and Lemma 2, makes it more likely that $\underline{\lambda_k} \geq 1$ (given that $V^j(\cdot, s_{LF}) < V^j(\cdot, s_{RF})$). Lastly, it decreases the difference between (8) and (6) and thus, from D.7 and Lemma 2, makes it less likely that $\underline{\mu_j} \geq 1$ (given that $V^k(\cdot, s_{RA}) < V^k(\cdot, s_{RF})$).

The larger c_k is, the more likely that it surpasses threshold $\underline{c_k}_k$ and thus, from D.5, that (a) is met. At the same time, it is less likely that it is below threshold $\bar{c}_{kj}$ and thus, from D.6, that (b) is met. It also increases the difference between (12) and (11) and thus, from D.8 and Lemma 2, makes it more likely that $\underline{\lambda_k} \geq 1$ (given that $V^j(\cdot, s_{LF}) < V^j(\cdot, s_{RF})$). Lastly, it increases the difference between (4) and (5) and decreases the difference between (8) and (6) and thus, from D.7 and Lemma 2, makes it less likely that $\underline{\mu}_j \geq 1$ (given that $V^k(\cdot, s_{RA}) < V^k(\cdot, s_{RF})$.)

The larger C_t are, the less likely that they are below their threshold $\bar{C}_{tj}$ and thus, from D.6, that (b) is met. It also increases the difference between (12) and (11) and thus, from D.8 and Lemma 2, makes it more likely that $\underline{\lambda}_k \geq 1$ (given that $V^j(\cdot, s_{LF}) < V^j(\cdot, s_{RF})$). Lastly, it increases the difference between (4) and (5) and thus, from D.7 and Lemma 2, makes it less likely that $\underline{\mu}_j \geq 1$ (given that $V^k(\cdot, s_{RA}) < V^k(\cdot, s_{RF})$).

Proof of Propositions 1 and 3

The result follows directly from Lemma 3.

Proof of Theorem 1

Larger e_1 makes it more likely that (a) holds and, thus, that $\lambda_k = 1$ is the dominant action for k at s. If (a) did not hold, larger e_1 still makes it less likely that $\underline{\mu}_j \geq 1$ and, hence, makes it more plausible that $\mu_j > \underline{\mu}_j$, inducing k to choose $\lambda_k = 1$. Thus, larger e_1 makes more likely pairwise outcomes in which $\lambda_k = 1$ and less likely those which involve otherwise.

Larger $\{e_k\}$ also make it more likely that (a) holds and, thus, that $\lambda_k = 1$ is the dominant action for k at s. If (a) did not hold, larger $\{e_k\}$ still make it less likely that $\underline{\mu}_j \geq 1$ and, hence, makes it more plausible that $\mu_j > \underline{\mu}_j$, inducing k to choose $\lambda_k = 1$. On the other hand, larger $\{e_k\}$ also make it less likely that $\underline{\lambda}_k \geq 1$ and, hence, makes $\lambda_k \geq \underline{\lambda}_k$ more plausible, inducing j to choose at least $\mu_j \in [0, 1]$ should (b) not hold. Thus larger $\{e_k\}$ tends to increase the likelihood of pairwise outcomes in which $\lambda_k = 1$, and decrease those in which $\lambda_k \neq 1$. In turn, this can induce at least $\mu_j \in [0, 1]$ when (b) does not hold and $\lambda_k \geq \underline{\lambda}_k$, or just $\mu_j = 1$ when (b) holds. Together, these increase the likelihood of outcomes $\mu_j = 1, \lambda_k = 1$, decreases that of $\mu_j = 1, \lambda_k = 0$ and of $\mu_j = 0, \lambda_k = 0$, but may increase or decrease the likelihood of $\mu_j \in [0, 1], \lambda_k = 1$.

Larger r_1 makes it more likely that (a) holds and, thus, that $\lambda_k = 1$ is the dominant action for k at s. It also makes it less likely that $\underline{\mu}_j \geq 1$ and, hence, more plausible that $\mu_j \geq \underline{\mu}_j$, inducing at least $\lambda_k \in [0, 1]$. Thus, to the extent that r_1 makes (a) more likely to hold, it increases the likelihood of $\lambda_k = 1$ and decreases that of $\lambda_k \in [0, 1]$. But if (a) does not hold, r_1 increases the likelihood of $\lambda_k \in [0, 1]$. This is why there is an ambiguous effect on the likelihood of alliance by conquest. Similarly, larger r_1 makes it more likely that (b) holds and, hence, that $\mu_j = 1$ is the dominant action for j at s. It also make is $\underline{\lambda}_k \geq 1$ more likely and, hence, makes $\lambda_k \leq \underline{\lambda}_k$ more plausible, inducing j to choose at most $\mu_j \in [0, 1]$ if (b) did not hold. Thus, to the extent that r_1 makes (b) more likely to hold, it makes $\mu_j = 1$ more likely and $\mu_j \in [0, 1]$ less likely. But if (b) did not hold, it make $\mu_j \in [0, 1]$ more likely and $\mu_j = 1$ less likely. This is why r_1 has an ambiguous effect on the likelihood of alliance with unrest. (To the extent that r_1 makes (a) and (b) more likely to hold, it increases the likelihood of peaceful alliance, and decreases that of no alliance.)

Larger $\{r_k\}$ makes it more likely that (a) holds and, thus, that $\lambda_k = 1$ is the dominant action for k at s. It also makes $\underline{\mu_j} \geq 1$ less likely and, hence, more plausible that $\mu_j \geq \underline{\mu_j}$ more likely, inducing at least $\lambda_k \in [0, 1]$. But since $\{r_k\}$ also make (a) more likely, it tends to increase $\lambda_k = 1$ and decrease $\lambda_k \in [0, 1]$. Furthermore, $\{r_k\}$ increases the likelihood that $\underline{\lambda_k} \geq 1$ and, hence, less plausible that $\lambda_k \geq \underline{\lambda_k}$, inducing at most $\mu_j \in [0, 1]$ if (b) did not hold. $\{r_k\}$ do not affect (b), and only tends to make $\mu_j \in [0, 1]$ more likely through increasing the likelihood of $\underline{\lambda_k} \geq 1$.

Larger c_k makes it more likely that (a) holds and, thus, that $\lambda_k = 1$ is the dominant action for k at s. If (a) did not hold, it makes it likely that at least $\lambda_k \in [0, 1]$, since it decreases the likelihood of $\underline{\mu_j} \geq 1$. To the extent that c_k makes (a) more likely, it tends to increase the likelihood of outcomes involving $\lambda_k = 1$ and tends to decrease those involving $\lambda_k \in [0, 1]$ and $\lambda_k = 0$. However, larger c_k also makes (b) less likely to hold and $\underline{\lambda_k} \geq 1$ more likely, inducing at most $\mu_j \in [0, 1]$. Thus, c_k also decreases the likelihood of outcomes involving $\mu_j = 0$. Thus, the effect on peaceful alliance and no alliance is ambiguous, while it decreases the likelihood of alliance by conquest, but decreases that of alliance with unrest.

Larger C_t make it less likely that (b) holds and, hence, that $\mu_j = 1$ is a dominant action for j at s. Also, it makes $\underline{\lambda_k} \geq 1$ more likely and, hence, less plausible that $\lambda_k \geq \underline{\lambda_k}$, inducing at most $\mu_j \in [0, 1]$. Thus, C_t makes less likely outcomes involving $\mu_j = 1$ and more likely those involving $\mu_j \in [0, 1]$ and $\mu_j = 0$.

Proof of Proposition 2

It is straightforward to see that Proposition 3 gives rise to four types of 'benchmark' equilibra when each set of conditions hold at every state s. One is when, at every s, the conditions in (1) hold and thus, j always chooses $\mu_j = 1$ and every k chooses $\lambda_k = 1$. We call this equilibrium as one of peaceful consolidation, in which every player, whenever drawn, accepts the proposal to join (or stay) in the coalition. The grand coalition is thus formed and remains intact. When the condition in (2) holds at every s, such that j always chooses $\mu_j = 1$, and every k chooses $\lambda_k \in [0, 1)$, then there is always some probability of fighting, but the coalition always wins. Thus, every one joins the coalition, albeit by conquest. We call this consolidation by conquest. When the condition in (3) holds at every s, such that j always chooses $\mu_j \in [0, 1)$, and every k chooses $\lambda_k = 1$, then every responder (peacefully) joins the coalition, but because there is always some probability of rebellion, the grand coalition is not sustainable. We call this a fragmented polity. Lastly, when the condition in (4) holds at every s, such that j always chooses $\mu_j = 0$ and every k chooses $\lambda_k = 0$, then no alliance is ever made, and each player remains its own singleton coalition. We call this equilibrium as one of independent territories.

Proof of Proposition 4

We want to provide conditions such that $\mu_j = 1$ is (uniquely) chosen at each s, and not $\mu_j \in [0, 1]$. We know that $\mu_j = 1$ for any $\lambda \in [0, 1]$ if (b) holds. Otherwise, if (b) does not hold, then $\mu_j = 1$ if $\lambda_k > \underline{\lambda_k}$. Now, in turn, the latter implies that if (a) holds (such that $\lambda_k = 1 \forall \mu_j \in [0, 1]$), $\underline{\lambda}_k$ must be less than one. (Otherwise, $\lambda_k > \underline{\lambda}_k$ cannot be met.) If (a) does not hold, then either $\lambda_k = 1$ (if $\underline{\mu}_j < 1$), or $\lambda_k \in [0, 1]$ (if $\overline{\mu}_j \geq 1$). Thus, if $\underline{\mu}_j < 1$, it must be that $\underline{\lambda}_k < 1$. We cannot have $\underline{\mu}_j \geq 1$, because $\overline{\lambda}_k$ can be $\lambda_k \leq \underline{\lambda}_k$ (in the extreme, λ_k can be zero).

But we also know that if (a) and (b) both hold, that $\mu_j = 1$ is the dominant action of j. One can also obtain $\mu_j = 1$ if (a) holds, but not (b), if $\underline{\lambda}_k < 1$. Thus, for $\mu_j = 1$ at each s, it must be that at each s, either: (1) (a) and (b) hold; (2) (a) holds, (b) does not hold, and $\underline{\lambda}_k < 1$; (3) (a) does not hold, (b) holds, and $\underline{\mu}_j < 1$; and (4) (a) does not hold, (b) holds, and $\underline{\mu}_j \geq 1$. But (3) and (4) can be combined into: (5) (a) does not hold, (b) holds. Thus, (1), (2) and (5) together imply that necessary and sufficient for $\mu_j = 1$ is that either (b) holds or, if (b) does not hold, that $\underline{\lambda}_k < 1$.

Proof of Theorem 2

From Proposition 4, we know that the relevant conditions are (b) and whether $\underline{\lambda}_k < 1$. From Corollary 2, variables that make it likely for (b) to hold are (small) c_k and C_t, and (large) r_1. Variables that make it likely for $\underline{\lambda}_k < 1$ are (large) $\{e_k\}$, and (small) r_1, $\{r_k\}$, c_k, C_t. From the foregoing, one can infer that small C_t^* and c_k^* make it likely for consolidation to happen since, in the first place, they make it likely that (b) holds at each state s. That is, if (b) holds for the largest possible costs C_t^*, c_k^*, they also hold for lesser costs. In the second place, even if (b) did not hold at some of all states (e.g. when r_1, C_t, or c_k are too small in that state), they make it more likely that $\underline{\lambda}_k < 1$ in those states (or, equivalently, less likely that $\underline{\lambda}_k \geq 1$.)

Similarly, large $\{e_k\}$ and small $\{r_k\}$ monotonically increase the likelihood of consolidation in that they make it more likely that $\underline{\lambda}_k < 1$, although they are thus only relevant if c_k or C_t are not sufficiently small, or r_1 not sufficiently large, such that (b) does not hold.

In contrast, the effect of r_1 is non-monotonic. The foregoing suggests that increasing r_1 makes (b) more likely to hold and, hence, increase the likelihood of consolidation. However, while decreasing r_1 thus makes (b) less likely to hold, it also makes $\underline{\lambda}_k < 1$ more likely and thereby also increase the likelihood of consolidation. For these to both be true, there must be some $r_{1j}^0 < r_{1j}^*$, where r_{1j}^0 is the smallest possible threshold $\underline{r}_{1j}$, and r_{1j}^* the largest, for j across all states, such that in the range $r_1 \in [0, r_{1j}^0)$, the likelihood of consolidation is decreasing in r_1 at all states, while in the range $r_1 \in [r_{1j}^0, r_{1j}^*]$, it is increasing in r_1. For $r_1 \in (r_{1j}^*, \infty)$, given that C_t and C_k are at or below their thresholds, r_1 has no further effect on the likelihood of consolidation, since (b) already holds and $\mu_j = 1$ the dominant action for j, and the likelihood of consolidation is therefore one.

Proof of Corollary 1

See discussion in text and the proof of theorem 2.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s00199-024-01583-8.

References

Abramson, S.: The economic origins of the territorial state. Int. Organ. **71**(1), 97–130 (2017)
Acemoglu, D., Robinson, J.A.: Weak, despotic, or inclusive? how state type emerges from state versus civil society competition. Am. Political Sci. Rev. **117**(2), 407–420 (2023)
Acemoglu, D., Egorov, G., Sonin, K.: Coalition formation in non-democracies. Rev. Econ. Stud. **75**(4), 987–1009 (2008). (**10**)
Allmand, C.: Henry V. Yale University Press, New Haven and London (1992)
Andersen, T.B., Jensen, P.S., Skovsgaard, C.V.: The heavy plow and the agricultural revolution in Medieval Europe. J. Dev. Econ. **118**, 133–149 (2016)
Anderson, P.: Lineages of the Absolutist State. Verso, London (1974)
Bachrach, B.S., Bachrach, D.S.: Warfare in Medieval Europe c.400-c.1453, London and New York: Routledge, (2017)
Barlow, F.: The Feudal Kingdom of England 1042-1216, 5th edn. Longman, New York (1999)
Barlow, F.: William Rufus. Yale University Press, New Haven (2000)
Bartlett, R.: England Under the Norman and Angevin Kings, 1075–1225. Oxford University Press, Oxford (2000)
Barzel, Y.: A Theory of the State: Economic Rights, Legal Rights, and the Scope of the State, Cambridge. CUP, UK (2002)
Bates, R., Greif, A., Singh, S.: Organizing violence. J. Conflict Resolut. **46**, 599–628 (2002)
Bates, R.H.: Prosperity & Violence: The Political Economy of Development. W.W. Norton & Company, New York (2001)
Bates, R.H., Apart, W.T.F.: State Failure in Late-Century Africa. Cambridge University Press, Cambridge (2008)
Bates, R.H.: The Development Dilemma, Princeton. Princeton University Press, NJ. (2017)
Beeler, J.: Warfare in Feudal Europe, 720–120. Cornell University Press, Ithaca and London (1971)
Bisson, T.N.: The 'Feudal Revolution': reply. Past & Present **155**, 208–225 (1997)
Bisson, T.: The Crisis of the Twelfth Century: Power, Lordship, and the Origins of European Government. Princeton University Press, New Jersey (2009)
Blaydes, L.: State building the Middle East Middle. Annu. Rev. Polit. Sci. **20**, 487–504 (2017)
Bloch, M.: Feudal Society, Vol. 1, Chicago: University of Chicago Press, 1961. translated by L.A. Mayon
Bloch, M.: Feudal Society, Vol. 2, Chicago: University of Chicago Press, 1964. translated by L.A. Mayon
Brown, E.A.R.: The tyranny of a construct: feudalism and historians of medieval Europe. Am. Hist. Rev. **79**(4), 1063–1088 (1974)
Brown, A.R.: A list of castles, 1154–1216. The English Historical Review **74**(291), 249–280 (1959)
Cappelen, C., Hariri, J. G.: Tracing the Origins of the Early Modern State: Introducing the Castles Data, September 2022. Working Paper
Carpenter, D.: The Struggle for Mastery. Penguin, London (2003)
Chaney, E., Blaydes, L.: The feudal revolution and Europe's rise: political divergence of the Christian and Muslim worlds before 1500 CE. Am. Political Sci. Rev. **107**(1), 16–34 (2013)
Collins, R.: Charlemagne. Toronto University Press, Toronto (1998)
Cox, G.W., North, D.C., Weingast, B.R.: "The Violence Trap: A Political-Economic Approach to the Problems of Development," February (2015). Working Paper, Hoover Institution, Stanford University
Cuttler, S.H.: The Law of Treason and Treason Trials in Later Medieval France. Cambridge University Press, Cambridge (1981)
Desierto, D.A., Hall, J., Koyama, M.: Magna Carta. GMU Working Paper in Economics No. 23–34 (2023)

Van Dam, R.: "Merovinigian Gaul and the Frankish Conquests," in Paul Fouracre, ed., The New Cambridge Medieval History, Cambridge University Press Cambridge, pp. 193–231 (2005)
Duby, G.: The Threee Orders: Feudal Society Imagined, Chicago: University of Chicago Press. Translated by Arthur Goldhammer (1981)
Ertman, T.: Birth of Leviathan. Cambridge University Press, Cambridge (1997)
Fearon, J.D., Laitin, D.D.: Ethnicity, insurgency, and civil war. Am. Political Sci. Rev. **2**, 75–90 (2003)
Ferández-Villaverde, J., Koyama, M., Lin, Y., Sng, T.-H.: The fractured land hypothesis. Q. J. Econ. **138**(2), 1173–1231 (2023)
Ganshof, F.L.: Feudalism, London: Longman, (1951). translated by Philip Grierson
Gennaioli, N., Voth, H.-J.: State capacity and military conflict. Rev. Econ. Stud. **82**(4), 1409–1448 (2015)
Gillingham, J.: The Angevin Empire, 2nd edn. Arnold Publishers, London (2001)
Gravett, C.: Medieval Siege Warfare. Osprey Publishing Ltd, Oxford (1990)
Greif, A., Rubin, J.: "Endogenous Political Legitimacy: The Tudor Roots of England's Constitutional Governance," Journal of Economic History, 2024, Forthcoming
Grossman, H.I.: "Make us a king": anarchy, predation, and the state. Eur. J. Polit. Econ. **18**(1), 31–46 (2002)
Grzymala-Busse, A.: Beyond war and contracts: the medieval and religious roots of the European state. Annu. Rev. Polit. Sci. **23**(1), 19–36 (2020)
Hintze, O.: Military Organization and the Organization of the State. In: Gilbert, F. (ed.) The historical essays of Otto Hintze, pp. 178–215. Oxford University Press, Oxford 1906 (1975)
Holland, T.: The Forge of Chistendom. Doubleday, New York (2008)
Hollister, C.: Warren, Henry I. Yale University Press, New Haven (2001)
Huning, T.R., Wahl, F.: You reap what you know: appropriability and the origin of European states. Eur. J. Polit. Econ. **79**, 102432 (2023)
James, E.: The Origins of France: From Clovis to the Capetians 500–1000, Houndsmills. McMillan Press Limited, Basingstoke (1982)
Johnson, N.D., Koyama, M.: States and economic growth: capacity and constraints. Explor. Econ. Hist. **64**(2), 1–20 (2017)
Kitamura, S., Lagerlöf, N.-P.: Geography and state fragmentation. J. Eur. Econ. Assoc. **18**(4), 1726–1769 (2019)
Koutsougeras, L.C.: Coalitions with limited coordination. Econ. Theor. **73**(2), 439–456 (2022)
Leon, G.: Feudalism, collaboration and path dependence in england's political development. British J. Political Sci. **50**(2), 511–533 (2020)
Levine, D.K., Modica, S.: Anti-malthus: conflict and the evolution of societies. Res. Econ. **67**(4), 289–306 (2013)
Levine, D.K., Modica, S.: Survival of the weakest: why the west rules. J. Econ. Behav. Organ. **204**, 394–421 (2022)
Marx, K., Engels, F.: The Communist Manifesto, New York: Penguin Classics, 1848. Translated by Samuel Moore and Introduced by Gareth Stedman
Mayshar, J., Moav, O., Pascali, L.: The origin of the state: land productivity or appropriability? J. Polit. Econ. **130**(4), 1091–1144 (2022)
Mayshar, J., Moav, O., Neeman, Z.: Geography, transparency, and institutions. Am. Political Sci. Rev. **111**(3), 622–636 (2017)
McKitterick, R.: Charlemagne: The formation of a European Identity. Cambridge University Press, Cambridge (2008)
Moore, B.: Social Origins of Dictatorship and Democracy: Lord and Peasant in the Making of the Modern World, Boston. Beacon Press, MA (1966)
Moselle, B., Polak, B.: A model of a predatory state. J. Law, Econ. Organ. **17**(1), 1–33 (2001)
Myerson, R.B.: The autocrat's credibility problem and foundations of the constitutional state. Am. Political Sci. Rev. **102**(1), 125–139 (2008)
North, D.C., Wallis, J.J., Weingast, B.R.: Violence and Social Orders: a conceptual framework for interpreting recorded human history. Cambridge University Press, Cambridge (2009)
Okada, A.: Dynamic bargaining with voluntary participation and externalities. Econ. Theor. **75**(2), 427–452 (2023)
Olson, M.: Dictatorship, democracy, and development. Am. Political Sci. Rev. **87**, 567–576 (1993)
Painter, S.: The Rise of the Feudal Monarchies, Ithaca. Cornell University Press, New York (1951)
Parker, G.: The "Military Revolution", 1560–1660-a Myth? J. Mod. Hist. **48**(2), 195–214 (1976)
Phillips, S.: Edward II. Yale University Press, New Haven and London (2011)

Pollack, A.F., Maitland, F.W.: The History of England Law, Before the Time of Edward I, vol. 2. Cambridge University Press, Cambridge (1895)
Power, D.: "French and Norman Frontiers in the Central Middle Ages," in Daniel Power and Naomi Standen, eds., Frontiers in Question: Eurasian Borderlands, 700-1700, Palgrave Macmillan, pp. 105–127 (1999)
Prestwich, M.: Edward I. Yale University Press, 1988, New Haven (1997)
Ray, D.: A Game-Theoretic Perspective on Coalition Formation. Oxford University Press, Oxford (2007)
Ray, D., Rajiv V.: "Chapter 5 - Coalition Formation," in H. Peyton Young and Shmuel Zamir, eds., Handbook of Game Theory with Economic Applications, Vol. 4 Elsevier, pp. 239–326 (2015)
Reynonds, S.: Fiefs and Vassals. Oxford University Press, Oxford (1994)
Riker, W.H.: The Theory of Political Coalitions. Yale University Press, New Haven (1962)
Rubin, M.: The Hollow Crown: A History of Britain in the Late Middle Ages. Allen Lane, London (2005)
Salter, A.W., Young, A.: The Medieval Constitution of Liberty: Political Foundations of Liberalism in the West. University of Michigan Press, Ann Arbor (2023)
Scheidel, W.: Escape from Rome, Princeton. Princeton University Press, NJ (2019)
Scott, J.C.: Against the Grain, Princeton. Princeton University Press, NJ. (2017)
Smith, A.: An Inquiry into the Nature and Causes of the Wealth of Nations. Clarendon Press, Oxford (1776)
Spruyt, H.: The Sovereign State and Its Competitors. Princeton University Press, Princeton (1994)
Strayer, J.: On the Medieval Origins of the Modern State, Princeton. Princeton University Press, New Jersey (1970)
Tilly, C.: Reflections on the History of European State-Making. In: Tilly, C. (ed.) The Formation of Nation States in Western Europe, pp. 3–84. Princeton University Press Princeton, New Jersey (1975)
Tilly, C.: Coercion, Capital, and European States, AD 990–1990. Blackwell, Oxford (1992)
Turner, R.V., John, K.: England's Evil King?, Stroud. Tempus Publishing Limited, Gloucestershire (2005)
Ullman, W.: Principles of Government and Politics in the Middle Ages. Methuen and Co., Limited, London (1961)
Wallace-Hadrill, J.M.: The Long-Haired Kings. University of Toronto Press, Toronto (1982)
Weber, M.: Economy and Society. Bedminster, 1922, New York (1968)
Weingast, B.R.: "The Medieval Expansion of Long-Distance Trade: Adam Smith on the Towns' Escape from the Violent, Feudal Equilibrium," in "The Oxford Handbook of Institutions of International Economic Governance and Market Regulation," Oxford University Press, (2020)
White, L.: Medieval Technology and Social Change. Oxford University Press, Oxford (1962)
Wickham, C.: Framing the Middle Ages. Oxford University Press, Oxford (2005)
Wickham, C.: The Inheritance of Rome. Viking, New York (2009)
Young, A.T.: Hospitalitas: barbarian settlements and constitutional foundations of medieval Europe. J. Inst. Econ. **14**(4), 715–737 (2018)